WORKBOOK

Student Notes and Problems

PHYSICS 11
British Columbia

CASTLE ROCK
RESEARCH CORP

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Dedicated to the memory of Dr. V. S. Rao

STUDENT NOTES AND PROBLEMS WORKBOOKS

Student Notes and Problems (SNAP) workbooks are a series of support resources in mathematics for students in grades 3 to 12 and in science for students in grades 9 to 12. SNAP workbooks are 100% aligned with curriculum. The resources are designed to support classroom instructions and provide students with additional examples, practice exercises, and tests. SNAP workbooks are ideal for use all year long at school and at home.

The following is a summary of the key features of all SNAP workbooks.

UNIT OPENER PAGE

- Summarizes the curriculum outcomes addressed in the unit in age-appropriate language
- Identifies the lessons by title
- Lists the prerequisite knowledge and skills the student should know prior to beginning the unit

LESSONS

- Provide essential teaching pieces and explanations of the concepts
- Include example problems and questions with complete, detailed solutions that demonstrate the problem-solving process

NOTES BARS

- Contain key definitions, formulas, reminders, and important steps or procedures
- Provide space for students to add their own notes and helpful reminders

PRACTICE EXERCISES

- Include questions that relate to each of the curriculum outcomes for the unit
- Provide practice in applying the lesson concepts

REVIEW SUMMARY

- Provides a succinct review of the key concepts in the unit

PRACTICE TEST

- Assesses student learning of the unit concepts

ANSWERS AND SOLUTIONS

- Demonstrate the step-by-step process or problem-solving method used to arrive at the correct answer
- *Answers and solutions for the **odd-numbered questions** are provided in each student workbook.*
- *A SNAP Solutions Manual* that contains the full answers and complete solutions for all questions is available for download from the book's product page on the Castle Rock Research website.

Note to Students

Answers to problems are provided with the appropriate number of significant digits. To arrive at similar answers, follow this procedure when performing calculations:

- Use as many significant digits as possible when entering values into your calculator.
  E.g., use $9.81\,\text{m/s}^2$ versus $9.8\,\text{m/s}^2$ for the acceleration due to gravity.
- Retain all digits in your calculator for intermediate steps before reaching your final answer.

$$\begin{aligned}\text{E.g.,}\quad \vec{F} &= m\vec{a}\\ &= 10.2\ \text{kg} \times 9.81\ \text{m/s}^2\\ &= 100.062\ \text{N}\end{aligned}$$

- If adding or subtracting, retain the same number of decimal places as the measured number with the smallest number of decimal places.

$$\begin{aligned}\text{E.g.,}\quad & 6.48\\ & +98.2\\ & \overline{104.68\ \text{m} \rightarrow 104.7\ \text{m}}\end{aligned}$$

- If multiplying or dividing, round off to the correct number of significant digits in your final answer—not in the intermediate steps.

$$\begin{aligned}\text{E.g.,}\quad W &= \vec{F}d\\ &= 100.062\ \text{N} \times 1.4\ \text{m}\\ &= 140.0868\ \text{J}\\ &= 1.4 \times 10^2\ \text{J}\end{aligned}$$

Please note: Intermediate steps presented in solutions in this book do not show all of the digits that were retained in the calculator. This is done to make the solutions easier to follow when reading through them.

E.g., In the *F* = *ma* example above, the answer is used to calculate the work done. Finding the force is an intermediate calculation. The solution will read as follows:

$$\begin{aligned}\vec{F} &= m\vec{a}\\ &= 10.2\ \text{kg} \times 9.81\ \text{m/s}^2\\ &= 100\ \text{N}\end{aligned}$$

$$\begin{aligned}W &= \vec{F}d\\ &= 100\ \text{N} \times 1.4\ \text{m}\\ &= 140\ \text{J}\\ &= 1.4 \times 10^2\ \text{J}\end{aligned}$$

At most, in intermediate steps, one additional digit may be presented beyond what is significant. This may appear in some of the solutions shown.

In the end, no matter which approach is used to solve a problem, final answers will be different in the last digit—the digit that requires estimation and rounding. This is acceptable, since the final digit contains uncertainty.

If two answers are presented that are different in the final digit, both answers should be considered correct solutions.

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INTRODUCTION TO PHYSICS

When you are finished this unit, you should be able to…

- Distinguish physics from other related fields of science
- Apply the concepts of accuracy and precision to experimental procedures and data: significant figures; uncertainty; scientific notation
- Plot linear relationships given a physical model (e.g., uniform motion, resistance)
- Calculate the slope of a line of best fit, including significant figures and appropriate units

Lesson	Page	Completed on
1. The Nature of Physics	2	
2. Significant Digits and Scientific Notation	4	
3. Graphing	7	
Answers and Solutions	at the back of the book	

Lesson 1 THE NATURE OF PHYSICS

NOTES

The word *physics* comes from a Greek word meaning *of nature*.

Physics is the study of matter and energy, and of their interactions. Of all the natural sciences, physics encompasses the greatest range of topics, from the properties of subatomic particles to the movement of stars and planets. Matter, energy, sound, light, and many other topics all make up what is known as the broad field of physics. Some of these subtopics are:

- Mechanics—the study of energy and force and their effects on objects.
  - Kinematics—the study of *how* objects move. Kinematics describes how to calculate an object's motion, such as its speed or acceleration.
  - Dynamics—the study of *why* objects move due to the effects of various forces on objects. It includes areas such as gravity, momentum, and Sir Isaac Newton's laws of motion.
- Thermodynamics—the study of heat, its effects, and its conversion to and from other forms of energy.
- Wave motion—the study of transmission of energy from place to place using a variety of wave types.
- Optics—the study of the behaviour of light. This includes how light moves, how it is transmitted, how it is transformed, and how it is perceived.
- Modern physics—the study of advanced topics in physics based on relatively recent discoveries, ideas, and technologies.
  - Relativity—the branch of physics that explains how different reference frames affect how space and time are perceived.
  - Nuclear physics—a modern branch of physics that deals with the structure of the atomic nucleus.
  - Quantum mechanics—the study of matter and energy at the atomic and sub-atomic level. There are certain things that Newtonian mechanics (see above) fails to explain: the movement of subatomic particles, condensed matter physics, and other advanced topics. Quantum mechanics offers a better way of addressing such issues, but it is notoriously complicated.
- Electromagnetism—the branch of physics that deals with electricity and magnetism, their interactions, and their effects.

The work of physicists is critical to the development of new technologies. Today, physicists are found in all manner of fields: academia, medicine, telecommunications, computers, environmental management, space science, industrial design, etc. Here are just a few of the things that physicists are working toward:

- Fusion power—power generated by fusion reactions. When two atomic nuclei are forced to combine into one heavier nucleus, an extraordinary amount of energy is generated. Physicists are working toward a way to use this process to generate power, which would be safer, cheaper, and more environmentally sound than fission (atom-splitting) reactors.

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- Nanotechnology—the application of technology on an extremely small scale—small enough for interaction with individual atoms. Nanotechnology could potentially revolutionize health-care, power generation, computer technology, and a host of other fields.
- Space exploration—the exploration of space through astronomy and the use of technology. Humans have already walked on the moon, lived in orbiting space stations, and sent machines to Mars to collect data. The next challenge is to create a machine that can return to Earth from Mars, bringing physical samples of rock, soil, and atmosphere that can be studied. A manned spaceflight to Mars may even be possible in this century.

Physics encompasses the entire physical universe, and is thus the basis of all other natural sciences (biology, chemistry, etc.). Ernest Rutherford, one of the world's first nuclear physicists, once said, "In science there is only physics; all the rest is stamp collecting." However, while physics may lie at the heart of the other sciences, it is not the best way to describe chemical or biological processes. Becoming an expert at making bricks does not make one an expert at building houses out of them; likewise, a physicist may know all the forces and particles that comprise an atom, but not know its effect on the human body or its place in a complex molecule.

The truth is that the sciences are becoming more and more dependent on each other, and each plays a key role in understanding nature and the universe. It is increasingly common for scientists to take on interdisciplinary roles. Biochemists, for example, study the effects of chemistry upon the human body, and can use this knowledge to develop medicines. Biophysicists develop machines used to study the human body (MRIs, X-Rays, etc.), and research human and animal motion, genetics, and a host of other topics.

Physics is an attempt to describe the fundamental processes of the universe in quantifiable terms. This means that physics relies heavily on mathematics: calculus, algebra, geometry, and trigonometry. The idea is that physics can use math to predict real-world outcomes.

Because so many variables can affect the outcome of an experiment (temperature, friction, gravity, etc.), experiments in physics must be done very carefully to make sure that all possible factors are considered. Experiments are designed to minimize, or account for, outside influences. Data collection can involve any number of devices, from ordinary metre sticks to billion dollar particle accelerators.

Before delving into the study of physics, it is beneficial to clarify the use of:

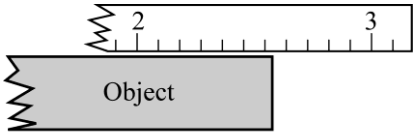
- significant digits
- scientific notation
- graphs

Lesson 2 SIGNIFICANT DIGITS AND SCIENTIFIC NOTATION

NOTES

Physics is reliant on the observation and measurement of data. Every measuring apparatus has a limit to how precisely it can measure. A consequence of this limitation is that the number of digits in our calculations known with certainty is also limited.

The digits that are known for certain, plus an estimated digit, are the significant digits.



Measurement = 2.64 units

It is clear that the object is between 2 and 3 units. By counting, one can also determine that it is just over 2.6 units. However, this ruler does not count hundredths. It can be seen that the object is not quite halfway between 2.6 and 2.7; therefore, the estimated length is 2.64. In summary, the value of the reading is 2.64 units (3 significant digits). Digits 2 and 6 are certain while 4 is an estimate.

ADDITION AND SUBTRACTION OF MEASUREMENTS

The sum or difference cannot be more precise than the least precise measurement.

e.g.: 2.71 m + 30.2 m = 32.9 m

Although both of these measurements have 3 significant digits, 2.71 m is measured to a hundredth of a metre, while 30.2 m is measured to a tenth of a metre. Tenths are less precise than hundredths. Therefore, the answer, 32.9, is given to the nearest tenth of a metre.

MULTIPLICATION AND DIVISION OF MEASUREMENTS

The rule is that the product or quotient has the same number of significant digits as did the measurement that had the fewest number of significant digits.

e.g.: 2.7 m × 21.9 m = 59 m^2

Any number (digit) 1–9 is a significant digit. Zeros may or may not be significant.

Zeros

- Zeros at the beginning of a quantity (measurement) are not significant (e.g. 0.027 m has 2 significant digits; zeros are not significant).

- Zeros that are sandwiched between significant digits are significant (e.g. 2.07 m has 3 significant digits).

- Zeros at the end of a quantity may or may not be significant (e.g. 20.0 m has 3 significant digits; 200 m has 1 significant digit).

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NOTES

In the case of 200 m, one cannot be sure whether or not the zeros are significant. The instrument could have read "200.012", and the value was rounded to the nearest metre. In this case, the zeros are significant. On the other hand, the instrument could have read "210.012", and the value was rounded to the nearest hundred metres. In this case, the zeros are not significant.

To avoid this complication, numbers are often expressed in *scientific notation*. Scientific notation is also used to express very large and very small numbers.

SCIENTIFIC NOTATION

First, establish whether the number is greater than 1 or less than 1. In the previous example, 200 m is greater than one.

Next, find out how many places are needed to shift the decimal to get a number between 1 and 10. In this case, the decimal needs to be shifted over 2 places. This gives a value of 2.00. To show the movement of the decimal, multiply the number by 10 to the power of the number of places the decimal was moved. The number would now be written as 2.00×10^2 m.

Now one can show how many significant digits the number has. If it only has 1 significant digit, it should be written as 2×10^2 m. If it has 2 significant digits, it should be written as 2.0×10^2 m. If it has 3, leave it as 2.00×10^2 m.

Note: 20.0 m has 3 significant digits. When a zero follows a decimal point, it is significant. It does not have to be there to show where the decimal point is, so the only reason for it being there is that it came off the measuring apparatus.

Large Numbers

Physicists often work with very large numbers, and scientific notation is usually the best way to express them. Astronomers frequently have to work with light speed, planetary distances, etc. For example, an astronomer is studying an object that is 234561000 km from Earth. It is lot easier to write (and to avoid errors) by expressing this in scientific notation:

234 561 000 km = 2.35×10^8 km (to 3 significant digits)

Small Numbers

Scientific notation also works for numbers under 1. For example, a physicist has calculated the weight of a tiny object as 0.0000125 g. Shift the decimal place to the right instead of to the left, until left with a number between 1 and 10—in this case, 1.25. Then, multiply the number by 10 to the *negative* power of the number of decimal places moved—in this case, 5. The number, expressed in scientific notation, is:

0.000 012 5 m = 1.25×10^{-5} m (to 3 significant digits)

PRACTICE EXERCISES

1. How many significant digits does each of the following measurements have?

a) 7.03 m

b) 0.075 m

c) 6.00 m

d) 200 m

e) 4.20×10^2 m

2. Write the following measurements in scientific notation.

a) 0.00340 m to 3 significant digits

b) 700 m to 2 significant digits

c) 559 m to 2 significant digits

d) 552 m to 2 significant digits

e) 4.05 m to 2 significant digits

3. Express the answer to the following addition problems in scientific notation.

a) 70.0 m + 2.32 m

b) 552 m + 7.1 m

c) 460 m + 29.8 m

4. Express the answer to the following multiplication problems in scientific notation.

a) 75 m × 0.82 m

b) 9.63 m × 1.9 m

c) 6.20×10^2 m × 20.0 m

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Lesson 3 GRAPHING

NOTES

A common process in physics, as well as other sciences, is to determine the relationship between variables. To do this, one variable is manipulated (the independent variable) to see how it affects another variable (the dependent variable) while keeping all other variables constant (the controlled variables). This demonstrates how one variable responds to a manipulation of another variable if all other variables are kept constant.

A good way to analyze this relationship is to plot a graph (the dependent variable vs. the independent variable). In the following example, the dependent variable is *A* and the independent variable is *B*. The units are hypothetical.

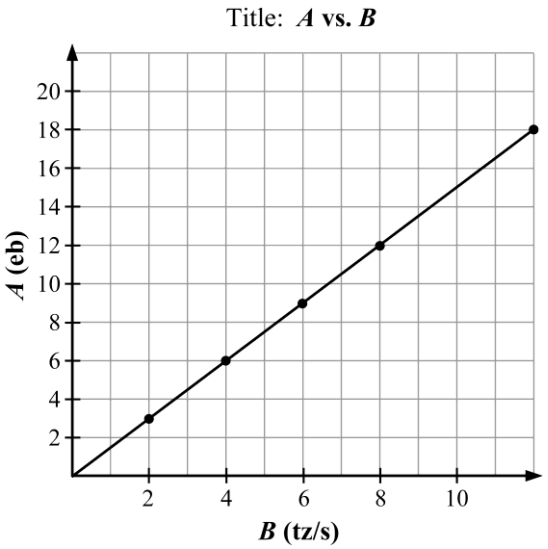
Units of *A* = eb
Units of *B* = tz/s
Data:

***A* (eb)**	***B* (tz/s)**
2.0	3.0
4.0	6.0
6.0	9.0
8.0	12.0
10.0	15.0
12.0	18.0

RULES FOR GRAPHING DATA

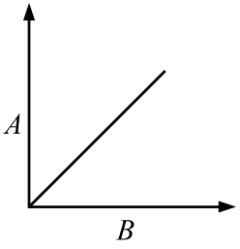
- Label the axes.
  - Independent variable should be placed on the *x*-axis, and the dependent variable on the *y*-axis.
  - Labels should include the units of the variables and any common exponents.
- Scale each axis.
  - Use a scale that will use as much of the graph as possible when the data is plotted on it.
  - Use a scale that is easy to read.
  - Make sure that the scale that you use is evenly spaced. That is, each division on the graph paper has the same value.
- Plot the data points and draw a smooth line of best fit.
- Give the graph a title. It always works to say "dependent variable vs. the independent variable."
- If a linear relationship (straight line) is suggested, use a straight edge to draw the line of best fit. If these points do not exactly line up, you will have some points above the line and some points below. This line is an approximation that is an average of all your data.

NOTES



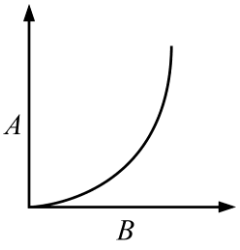
SHAPES OF GRAPHS

In the analysis of experimental data, it is helpful to be able to recognize the relationships that the shape of the line of best fit suggests.



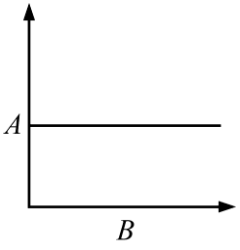
$A \propto B$ or $A = kB$

linear relationship



$A \propto B^2$ or $A = kB^2$

exponential relationship



no relationship

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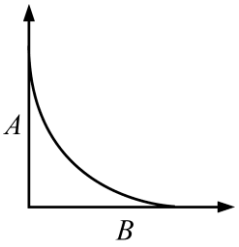


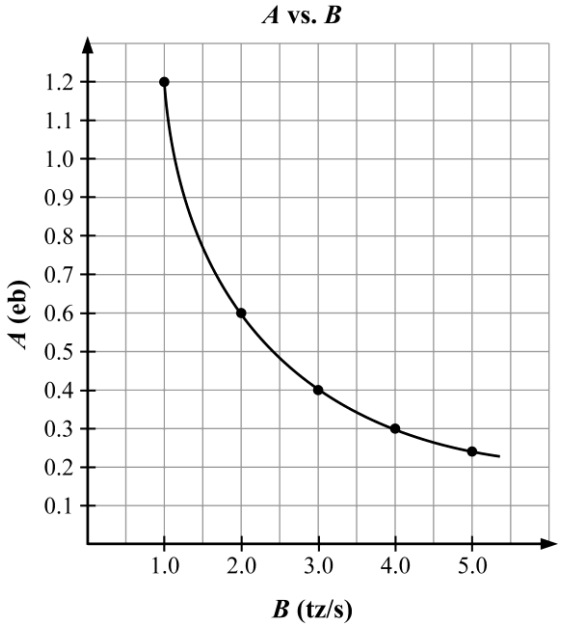
$$A \propto \frac{1}{B} \text{ or } A \propto \frac{1}{B^2}$$

$$A = \frac{k}{B} \text{ or } A = \frac{k}{B^2}$$

Is this an inverse relationship or inverse square relationship?

It is hard to tell if it is an inverse or inverse square relationship by just looking at the shape. Usually further analysis is necessary. This analysis involves curve straightening.

CURVE STRAIGHTENING



Data:

B (tz/s)	*A* (eb)
1	1.2
2	0.60
3	0.40
4	0.30
5	0.24

It is not known by looking at the above graph if it has a mathematical relationship of:

$$A \propto \frac{1}{B} \text{ or } A \propto \frac{1}{B^2}$$

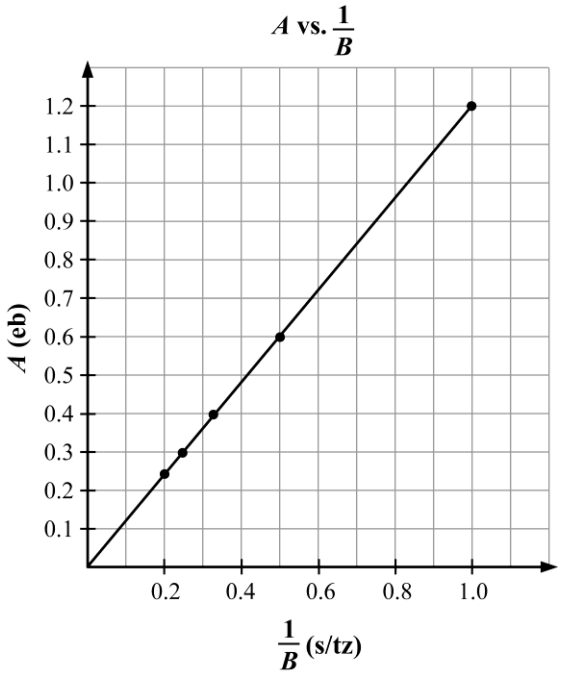
NOTES

However, by completing the following data table and drawing the graphs:

A vs. $\frac{1}{B}$ or A vs. $\frac{1}{B^2}$

B (tz/s)	A (eb)	$\frac{1}{B}$ (s/tz)	$\frac{1}{B^2}$ (s^2/tz^2)
1	1.2	1	1
2	0.60	0.50	0.25
3	0.40	0.33	0.11
4	0.30	0.25	0.063
5	0.24	0.20	0.040

Straight line



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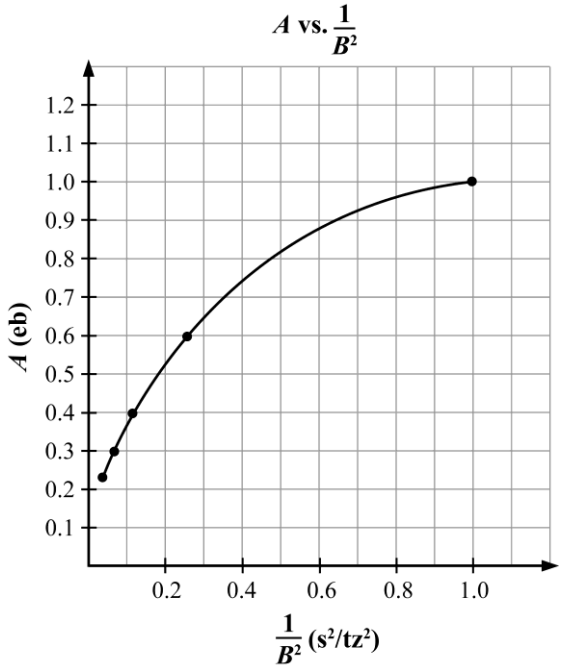
NOTES

Not a straight line

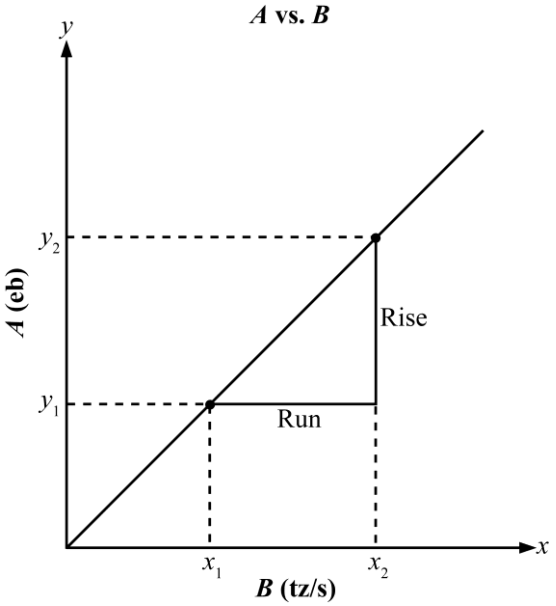


It is seen that A vs. $\frac{1}{B}$ gives a straight line. This indicates that $A \propto \frac{1}{B}$ and not $\propto \frac{1}{B^2}$.

NOTES

FINDING THE SLOPE

An important process in analyzing experimental data is to find the slope of a linear graph.



$$\text{slope} = \frac{\text{rise}}{\text{run}} \quad \text{or} \quad \frac{y_2 - y_1}{x_2 - x_1}$$

To find the slope, choose two points on the line of best fit.

- Choose these points as far apart as convenient, and so that you can read them easily.
- If your graph passes through the origin, by all means use it as one of the two points.
- Make sure you are using points on the graph. Do not use data points unless they are on the line.

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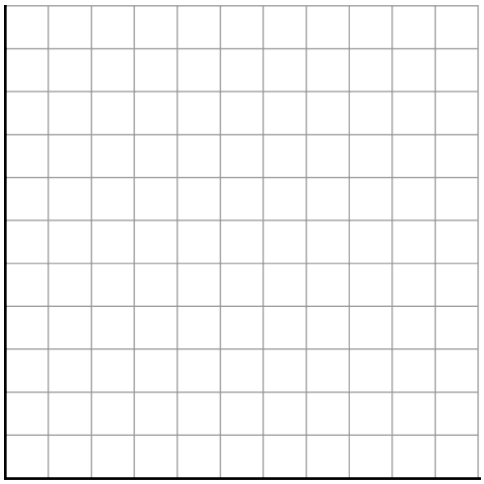
PRACTICE EXERCISES

1. A student collects the following data. She is attempting to find how *A* changes as she manipulates *B*.

***B* (tz/s)**	***A* (eb)**
3.00	0.151
6.00	0.310
9.00	0.448
12.0	0.600
15.0	0.750

a) Draw a graph showing the relationship between variable *A* and variable *B*.



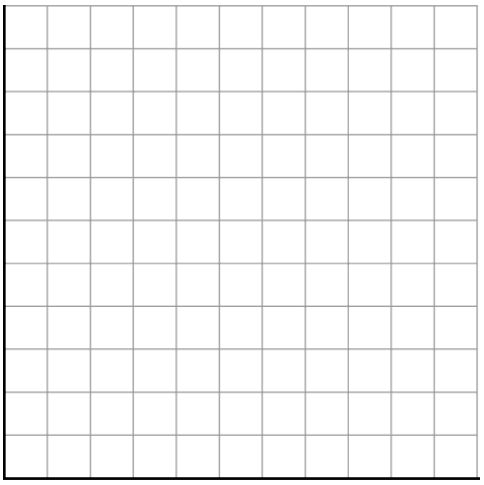
b) Find the slope of the graph.

c) What is the mathematical relationship between *A* and *B*?

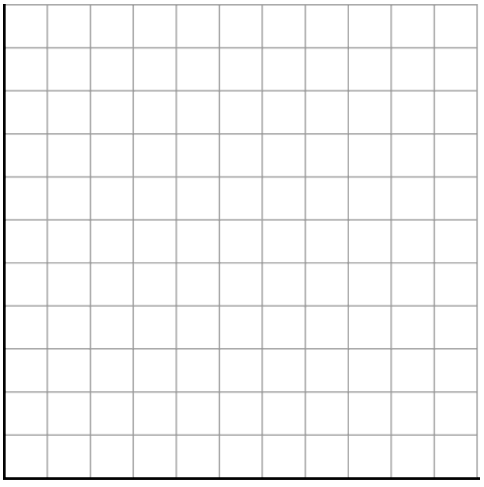
2. A student collects the following data. He is attempting to find how *A* changes as he manipulates *B*.

B ($\times 10^{-2}$ tz/s)	A (eb)	$\frac{1}{B}$ (s/tz)
2.1	4.0	
1.4	6.0	
1.1	8.0	
0.7	12.0	
0.35	24.0	

a) Draw a graph showing the relationship between variable *A* and variable *B*.



b) Complete the $\frac{1}{B}$ column in the above table, and draw the graph showing the relationship between variable *A* and $\frac{1}{B}$.



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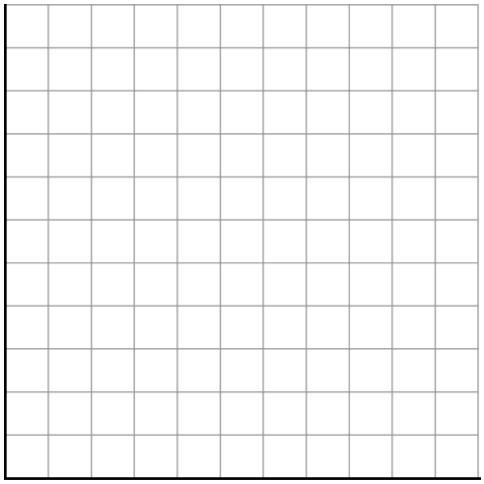
c) Find the slope of the graph you just drew.

d) What is the mathematical relationship between *A* and *B*?

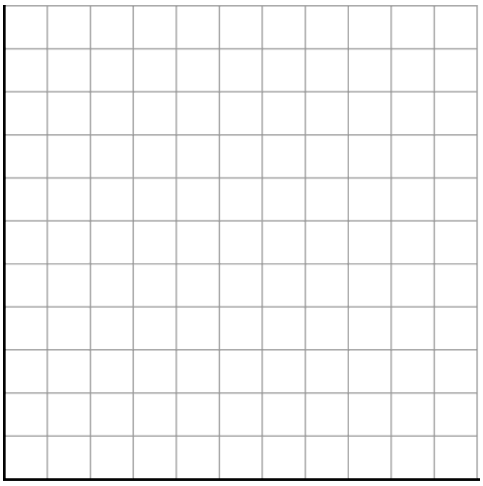
3. A student collects the following data. She is attempting to find how *A* changes as she manipulates *B*.

***B* (tz/s)**	***A* (eb)**	B^2 **(tz^2/s^2)**
1.5	2.25	
3.0	9.00	
4.5	20.25	
6.0	36.00	
7.5	56.25	
9.0	81.00	

a) Draw a graph showing the relationship between variable *A* and variable *B*.



b) Complete the B^2 column in the above table, and draw the graph showing the relationship between variable A and B^2.



c) Find the slope of the graph that you just drew.

d) What is the mathematical relationship between A and B?

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PREDICTING, ANALYZING, AND DESCRIBING MOTION

BIG IDEA

- An object's motion can be predicted, analyzed, and described.

As you begin this unit, you should consider the following questions…

- How can uniform motion and uniform acceleration be modelled?
- What are the limitations of free-body diagrams?
- How can the path of a projectile be changed?
- How does the launch angle affect the path of a projectile?
- How can the acceleration due to gravity be determined experimentally?

Lesson	Page	Completed on
1. Introduction to Kinematics	18	
2. Instantaneous Velocity and Speed	22	
3. Uniformly Accelerated Horizontal Motion in One Direction	32	
4. Freely Falling Objects	53	
5. Uniformly Accelerated Horizontal Motion in Two Directions along a Straight Line	63	
6 Uniformly Accelerated Vertical Motion in Two Directions along a Straight Line	81	
7. Projectile Motion	90	
Practice Test	95	
Answers and Solutions	at the back of the book	

When you are finished this unit, you are expected to know the following…

- The differences between **vector and scalar quantities**, including **addition and subtraction** and **right-angle triangle trigonometry**
- The analysis and description of **horizontal uniform and accelerated motion**, including graphical and quantitative analysis.
- The analysis and description of **projectile motion** in both **1 dimension and 2 dimensions**, **including vertical launch**, **horizontal launch**, and **angled launch**.

Lesson 1 INTRODUCTION TO KINEMATICS

NOTES

People observe objects in motion every day. A plane takes off from the airport. A student walks down the hall. Rain falls from the sky. Trees sway back and forth as they are pushed by the wind. The hands of the clock are in motion. How are all these motions described?

TERMINOLOGY

Before beginning the description of motion, some important terms should be defined:

- Displacement—the change in position of an object measured along a straight line. Displacement is a vector quantity and is measured in metres (m). The variable $\vec{d}$ is used for displacement and describes the total displacement including direction from some initial starting point.

The Greek letter *delta*, Δ, is a symbol meaning "change in".

- Velocity—the rate of change in an object's displacement with respect to time. Velocity is a vector quantity and is measured in metres per second (m/s).

$$\vec{v}_{ave} = \frac{\Delta \vec{d}}{\Delta t}$$

Rate of change is the rate that some quantity changes with respect to time.

The velocity found by this equation is an average over the time interval. This equation can also be used when you know the velocity is constant.

- Acceleration—the rate of change in an object's velocity with respect to time. Acceleration is a vector quantity and is measured in metres per second squared (m/s^2).

$$\vec{a}_{ave} = \frac{\Delta \vec{v}}{\Delta t}$$

A vector quantity has direction as well as magnitude.

Displacement, velocity, and acceleration are vector quantities.

The acceleration found by this equation is an average over the time interval. This equation can only be used when you know the acceleration is is constant. For the purposes of this text, the acceleration of an object will always be constant (uniform).

The quantities displacement, velocity, and acceleration are vector quantities. A vector quantity is a quantity that has direction as well as magnitude (size). This means that when describing displacement, velocity, or acceleration, magnitude is not enough. One must also indicate the direction. For example, if an object is moved 20 m, its new position is unknown. However, if it is moved 20 m east, its new position is known—its displacement is known. For now, the only directions that will be used are right and left or up and down since the motion that will be studied first is one dimensional. In one dimension, you only consider motion along one straight line. By convention, the directions up and right will be treated as positive, while left and down will be negative.

$$v_{ave} = \frac{\text{distance}}{\text{time}}$$

$$\vec{v}_{ave} = \frac{\text{displacement}}{\text{time}}$$

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There are three additional terms that are often used to describe motion:

- Distance—how far an object has moved measured along the path of motion. Distance is measured in metres (m). This differs from displacement in that it takes into account the total distance travelled and not just the final displacement from the initial position.
- Speed—the magnitude of velocity at a given time *t*. Speed, *v*, is measured in m/s.
- Average speed—the distance travelled divided by the total time of motion. Average speed is measured in m/s.

$$\text{average speed} = \frac{\text{distance}}{\text{time}}$$

or

$$v_{\text{ave}} = \frac{d}{t}$$

The quantities distance and speed are scalar quantities. A scalar quantity is a quantity that has magnitude (size) only. For example, if an object is moved 20 m, you know the distance moved but you do not know the displacement.

NOTES

A scalar quantity has only magnitude.

Distance and speed are scalar quantities.

Average velocity and average speed may also be denoted by $\bar{v}_{\text{av}}$ and v_{av}, respectively.

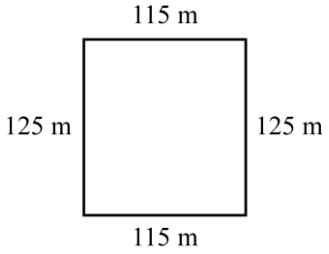
PRACTICE EXERCISES

1. A man walks 275 m right and then turns around and walks 425 m left.

a) What is the distance travelled by the man?

b) What is the displacement of the man?

2. A little girl takes her dog for a walk around a city block as described below.



a) What is the distance travelled by the girl and dog during the walk?

b) What is the displacement of the girl and dog after the walk?

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3. An object travels 11 m right and then turns around and travels 25 m left. If the total time of travel is 52 s, what is

a) the average speed of the object?

b) the average velocity of the object?

Lesson 2 INSTANTANEOUS VELOCITY AND SPEED

NOTES

The average velocity and average speed do not give any information about how fast an object is moving at any given time. It gives an average over time, as the term indicates.

Instantaneous velocity and instantaneous speed are the velocity and speed at some given instant in time.

NOTE: In this section, the terms velocity and speed will be used in place of instantaneous velocity and instantaneous speed.

KINEMATICS IN ONE DIMENSION

To better understand the mechanics of displacement, velocity and acceleration, they will be examined in one dimension. A body undergoing acceleration that changes with time is difficult to express mathematically. For simplicity, the following types of motion will be described:

Uniform motion is motion in which the velocity remains constant.

Uniformly accelerated motion is motion in which the acceleration remains constant.

- Uniform KINEMATICS—motion in which the velocity remains constant.
- Uniformly accelerated KINEMATICS— motion in which the acceleration remains constant.

These two types of motion will be analyzed graphically and mathematically so that an intuitive understanding of motion can be gained.

UNIFORM MOTION

Uniform motion is motion in which the velocity of an object remains constant (uniform). The displacement of the object will increase the same amount for each second that passes. That is, displacement is directly proportional to time. This can be expressed as:

$\vec{d} \propto t$

Since the velocity of the object is constant, the acceleration of the object must be zero.

The slope of a position-time graph represents the velocity of the object. The slope of a velocity-time graph represents the acceleration of the object. The area under a velocity-time graph is the displacement of the object.

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NOTES

Example

An object travels at a uniform velocity of 25.0 m/s right. What is the displacement of the object after 10.0 minutes?

Solution

$$\vec{v}_{ave} = \frac{\vec{d}}{t}$$
$$\vec{d} = \vec{v}_{ave} t$$
$$= (25.0 \text{ m/s})(6.00 \times 10^2 \text{ s})$$
$$= 1.50 \times 10^4 \text{ m right}$$

Example

After 3.0 h, an object's displacement is 2.60×10^2 km right. What is the average velocity of the object?

Solution

$$\vec{v}_{ave} = \frac{\vec{d}}{t}$$
$$= \frac{2.60 \times 10^2 \text{ km}}{3.0 \text{ h}}$$
$$= 87 \text{ km/h right}$$

Example

An object travels a distance of 5.0 m in 2.7 s. What is the average speed of the object?

Solution

$$v_{ave} = \frac{d}{t}$$
$$= \frac{5.0 \text{ m}}{2.7 \text{ s}}$$
$$= 1.9 \text{ m/s}$$

PRACTICE EXERCISES

Formula: $\bar{v}_{ave} = \dfrac{\vec{d}}{t}$

1. In 11.2 s an object is displaced 1.00×10^2 m to the right. What is the average velocity of the object?

2. An object is travelling at a constant velocity of 10.0 m/s left.

a) What distance will this object travel in 4.5 s?

b) What is the position of this object at 4.5 s?

3. An object is travelling at a constant velocity of 9.8 m/s right. How long will it take this object to travel a distance of 2.5 m?

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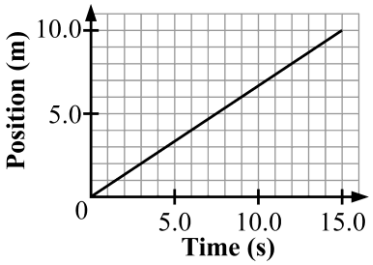
4. A man runs at an average velocity of 1.30 m/s right for 98.0 s, and then walks at an average velocity of 0.45 m/s right for 90.0 s. What is the average velocity of the man during his total time of travel?

5. An object is dropped from a height of 64.0 m. If the object takes 3.61 s to fall, what is the average velocity of the object?

6. Given the following position-time graph of an object moving right, find the velocity of the object at

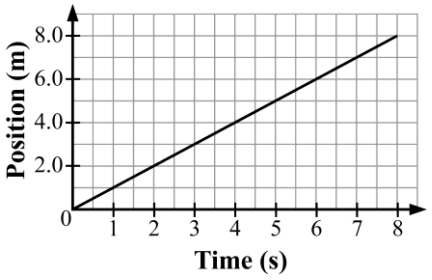


a) 8.0 s

b) 12.0 s

c) 1.0 s

7. Given the following position-time graph of an object moving right, find the



a) speed of the object at 6.0 s

b) velocity of the object at 6.0 s

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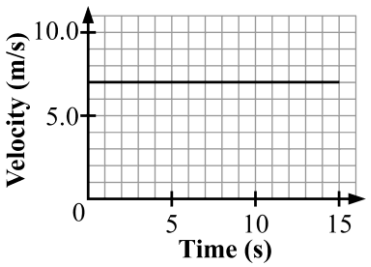
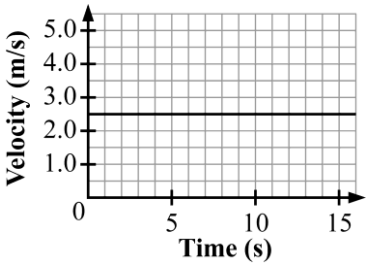
8. Given the following velocity-time graph of an object moving left, find the



a) displacement in 10 s

b) distance travelled in 10 s

9. Given the following velocity-time graph of an object moving left, find the



a) displacement in 5.0 s

b) distance travelled in 5.0 s

10. An object travels at a constant velocity of 5.0 m/s right for 31 min, and then 7.0 m/s in the same direction for 15 min. Calculate the average velocity.

11. An object travels at a constant velocity of 8.0 m/s right for 25 min, and then 5.0 m/s in the opposite direction for 15 min. Calculate the

a) average velocity

b) average speed

12. Object A travels at a constant velocity of 2.0 m/s right, and object B travels in the same direction at a constant velocity of 3.0 m/s. If object B starts 1.0 min after object A from the same position, how many seconds will it take object B to catch object A?

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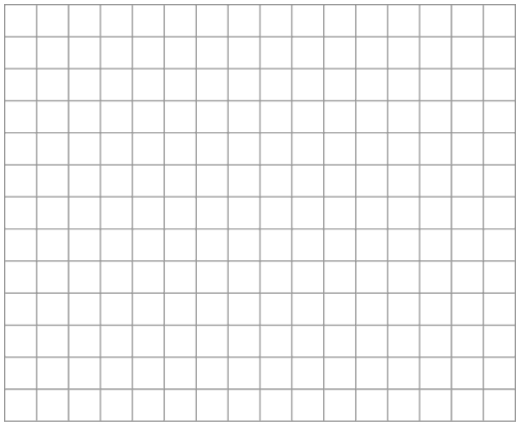
13. Given the following data obtained by moving an object at a constant velocity to the right

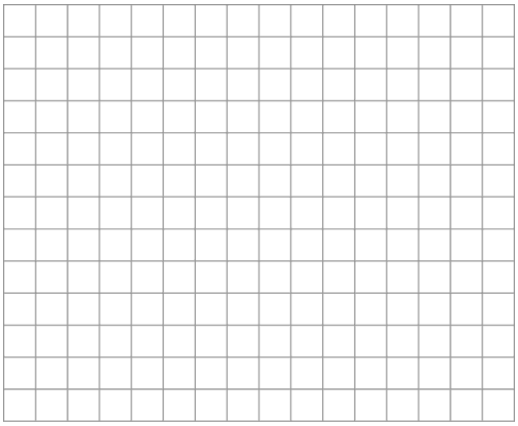
Time (s)	Displacement (m)	Displacement During Time Interval (m)	Average Velocity During Time Interval (m/s)
0	0		
0.10	0.012		
0.20	0.024		
0.30	0.035		
0.40	0.047		
0.50	0.060		
0.60	0.072		
0.70	0.085		
0.80	0.097		
0.90	0.108		
1.00	0.120		

a) complete the data table

b) draw a position-time graph



c) draw a velocity-time graph



d) Using the slope of your position-time graph, find the velocity of the object.

e) Using the area under your velocity-time graph, find the displacement by the object in 0.95 s.

14. The dots below represent the position of an object every 0.10 s as it moves right along a horizontal straight line.

$t = 0$

•• • • • • •

Analyze this motion by:

a) completing the following data table

Time (s)	**Displacement from t = 0 (m)**	**Displacement during time interval (m)**	**Average velocity during time interval (m/s)**
0	0		
0.10	0.016		
0.20	0.031		
0.30	0.046		
0.40	0.062		
0.50	0.076		
0.60	0.091		

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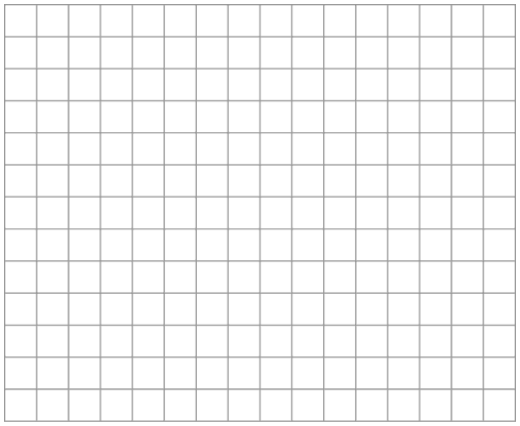
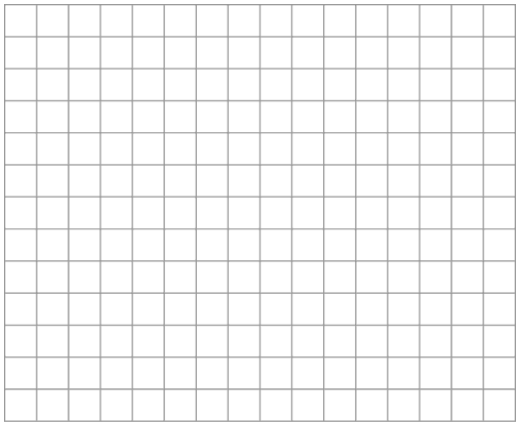
b) drawing a position-time graph



c) drawing a velocity-time graph



d) finding

i) the velocity of the object using your position-time graph

ii) the displacement of the object using your velocity-time graph

Lesson 3 UNIFORMLY ACCELERATED MOTION IN ONE DIRECTION

NOTES

In uniform accelerated motion, the acceleration is constant.

Acceleration is the rate of change in the velocity.

Average acceleration may also be denoted by $\vec{a}_{av}$.

Acceleration is the rate of change in velocity with respect to time.

$$\vec{a}_{ave} = \frac{\Delta \vec{v}}{\Delta t} \text{ or } \vec{a}_{ave} = \frac{\vec{v}_f - \vec{v}_i}{t}$$

where $\vec{v}_f$ = final velocity
$\vec{v}_i$ = initial velocity
t = time interval

In uniform accelerated motion, the acceleration of the object remains constant while the velocity and displacement both change over time. An object's velocity will increase at a linear rate. That is, for every second that passes, the velocity will increase by the same amount each second. The displacement is directly proportional to the square of the time travelled. This can be expressed as:
$\vec{v} \propto t$ and $\vec{d} \propto t^2$

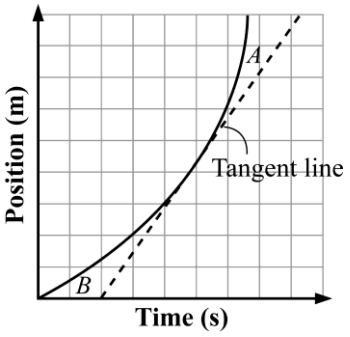
These relationships can be demonstrated through mathematical and graphical analysis.

INSTANTANEOUS VELOCITY (OR SPEED)

It has been demonstrated that the slope of a position-time graph represents the velocity of the object. For an object undergoing uniformly accelerated motion, you cannot determine the velocity of the object for a given time interval by using a single slope. However, it is possible to find the velocity at any given time (instantaneous velocity) by drawing a tangent line to the curve at that point, and then finding the slope of the line.

Suggestion:
Use a data table similar to those used in the problems on uniform motion.

Example



In drawing this line, estimate the angles A and B so that they are approximately equal.

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NOTES

In summary, graphical analysis is a powerful tool for examining an object undergoing uniform motion and uniform accelerated motion. It has been shown that:

- area under a velocity-time graph is the displacement
- slope of the position-time graph is the velocity
- slope of the velocity-time graph is the acceleration

DERIVING MOTION EQUATIONS

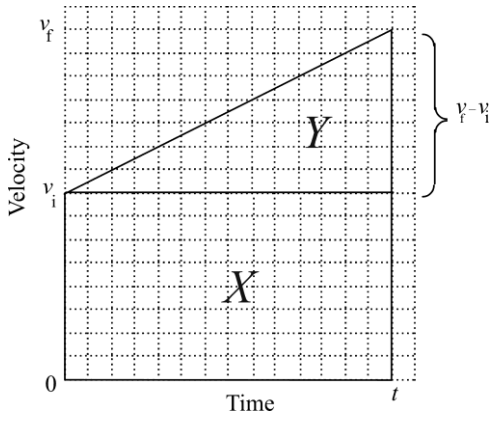
There are three additional equations that are used to describe uniform accelerated motion:

$$\vec{d} = \left(\frac{\vec{v}_f + \vec{v}_i}{2} \right) t$$

$$\vec{d} = \vec{v}_i t + \frac{1}{2} \vec{a} t^2$$

$$\vec{v}_f^{\,2} = \vec{v}_i^{\,2} + 2\vec{a}\vec{d}$$

These equations can be derived through graphical analysis.



The displacement is the area under the velocity-time graph. Therefore, the displacement is the area of the rectangle X plus the area of the triangle Y.

$$\text{area of rectangle X} = \vec{v}_i t$$

$$\text{area of rectangle Y} = \frac{1}{2} t(\vec{v}_f - \vec{v}_i)$$

$$= \frac{\vec{v}_f t}{2} - \frac{\vec{v}_i t}{2}$$

$$\text{total area of X and Y} = \vec{v}_i t + \left(\frac{\vec{v}_f t}{2} - \frac{\vec{v}_i t}{2} \right)$$

$$= \frac{\vec{v}_i t}{2} + \frac{\vec{v}_f t}{2}$$

$$= \left(\frac{\vec{v}_f + \vec{v}_i}{2} \right) t$$

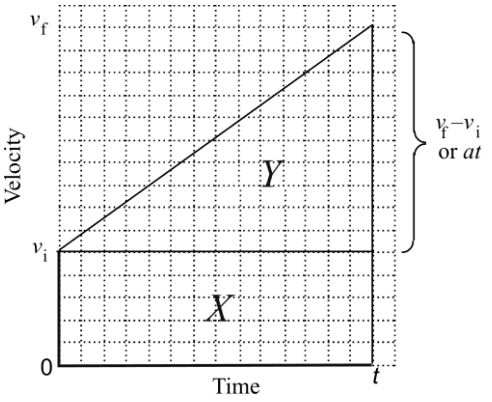
$$\vec{d} = \left(\frac{\vec{v}_f + \vec{v}_i}{2} \right) t$$

Describe the uniformly accelerated motion without knowing the acceleration:

$$\vec{d} = \left(\frac{\vec{v}_f + \vec{v}_i}{2} \right) t$$

NOTES

You can derive $\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$ in much the same way.



The displacement is the area under the velocity-time graph. The area is represented by a rectangle X and a triangle Y.

$$\text{area of rectangle X} = \vec{v}_i t \qquad \text{area of triangle Y} = \frac{1}{2}(\vec{a}t)(t) = \frac{1}{2}\vec{a}t^2$$

$$\text{total area of X and Y} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

$$\therefore \vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

Describe the uniformly accelerated motion without knowing the final velocity:

$\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$

The equation $\vec{v}_f^{\,2} = \vec{v}_i^{\,2} + 2\vec{a}\vec{d}$ is derived by combining equations $\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$ and $\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$.

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$

$$t = \frac{\vec{v}_f - \vec{v}_i}{\vec{a}}$$

Substitute t into the equation $\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$.

$$\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

$$\vec{d} = \vec{v}_i\left(\frac{\vec{v}_f - \vec{v}_i}{\vec{a}}\right) + \frac{1}{2}\vec{a}\left(\frac{\vec{v}_f - \vec{v}_i}{\vec{a}}\right)^2$$

$$= \left(\frac{\vec{v}_i\vec{v}_f}{\vec{a}} - \frac{\vec{v}_i^{\,2}}{\vec{a}}\right) + \left(\frac{\vec{v}_f^{\,2}}{2\vec{a}} - \frac{2\vec{v}_f\vec{v}_i}{2\vec{a}} + \frac{\vec{v}_i^{\,2}}{2\vec{a}}\right)$$

$$\vec{d} = \frac{\vec{v}_f^{\,2}}{2\vec{a}} - \frac{\vec{v}_i^{\,2}}{2\vec{a}}$$

$$2\vec{a}\vec{d} = \vec{v}_f^{\,2} - \vec{v}_i^{\,2}$$

$$\vec{v}_f^{\,2} = \vec{v}_i^{\,2} + 2\vec{a}\vec{d}$$

Describe the uniformly accelerated motion without knowing the time:

$\vec{v}_f^{\,2} = \vec{v}_i^{\,2} + 2\vec{a}\vec{d}$

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SUMMARY

NOTES

	$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$	✓	✓	✓		✓
$\vec{d} = \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right)t$	✓	✓		✓	✓
$\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$	✓		✓	✓	✓
$\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a}\vec{d}$	✓	✓	✓	✓	

Note that each equation has four of the possible five variables that can be used to describe uniform accelerated motion. In the mathematical analysis of uniform motion, the first thing that must be done is to establish the equation that will be used. This is done by listing all the given variables and the unknown variable. In this list there must be four variables (three known, and one unknown). From this list, you will note that one of the five variables listed in the summary is missing. The equation to use is the equation that is also missing this variable.

By convention, right and up are generally defined as the positive directions and left and down are considered the negative directions.

Remember:

Average velocity can be calculated using

$$\vec{v}_{ave} = \frac{\vec{d}}{t}$$

Under uniform accelerated motion, average velocity can also be calculated using

$$\vec{v}_{ave} = \frac{\vec{v}_f + \vec{v}_i}{2}$$

Example

An object that is initially travelling at a velocity of 7.0 m/s right accelerates uniformly to a velocity of 22.0 m/s right in a time of 1.7 s. Calculate the acceleration of the object.

Solution

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
7.0 m/s	22.0 m/s	?	X	1.7 s

NOTE: Since displacement is not a concern, it is correct to use the equation without the *d* variable.

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$
$$= \frac{22.0 \text{ m/s} - 7.0 \text{ m/s}}{1.7 \text{ s}}$$
$$= 8.8 \text{ m/s}^2 \text{ right}$$

NOTES

Example

An object accelerates right uniformly from rest in a time of 2.70 s. In this time, it travelled 20.0 m. What was the final velocity?

Solution

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
0	?	X	20.0 m	2.70 s

NOTE: Since acceleration is not a concern, it is correct to use the equation without the *a* variable.

$$\vec{d} = \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right)t$$

$$20.0\text{ m} = \left(\frac{\vec{v}_f}{2}\right)(2.70\text{ s})$$

$$\vec{v}_f = \frac{(2)(20.0\text{ m})}{(2.70\text{ s})}$$

$$\vec{v}_f = 14.8\text{ m/s right}$$

Example

An object accelerates uniformly from rest. If the acceleration was 2.00 m/s^2 left, what was the displacement when it reached a velocity of 1.00×10^2 km/h?

Solution

$$\vec{v}_f = 1.00\times10^2\text{ km/h}\times10^3\text{ m/km}\times\text{h}/3600\text{ s}$$
$$= -27.8\text{ m/s or }27.8\text{ m/s left}$$

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
0	–27.8 m/s	–2.00 m/s^2	?	X

NOTE: Since time is not a concern, it is correct to use the equation without the t variable.

$$\vec{v}_f^{\,2} = \vec{v}_i^{\,2} + 2\vec{a}\vec{d}$$

$$(-27.8\text{ m/s})^2 = 2(-2.00\text{ m/s}^2)\vec{d}$$

$$\vec{d} = \frac{(-27.8\text{ m/s})^2}{2(-2.00\text{ m/s})^2}$$

$$= -193\text{ m or }193\text{ m left}$$

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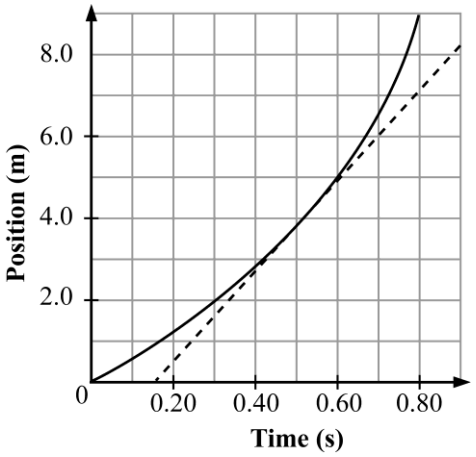
PRACTICE EXERCISES

Formulas:

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t} \qquad \vec{d} = \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right)t \qquad \vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v}_f^{\,2} = \vec{v}_i^{\,2} + 2\vec{a}\vec{d} \qquad \vec{v}_{ave} = \frac{\vec{v}_f + \vec{v}_i}{2}$$

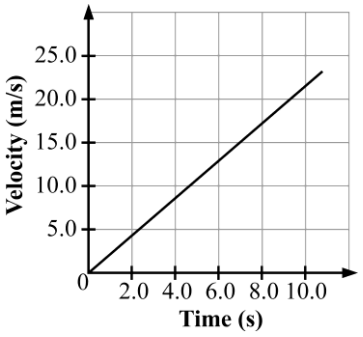
1. Given the following position-time graph of an object moving right, find the velocity of the object at



a) 0.40 s

b) 0.60 s

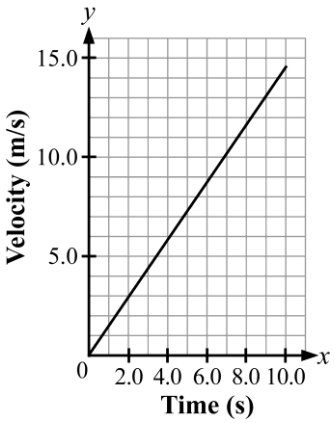
2. Given the following velocity-time graph for an object moving right, find



a) the acceleration of the object

b) the displacement in 7.0 s

3. Given the following velocity-time graph for an object moving right, find



a) the acceleration of the object

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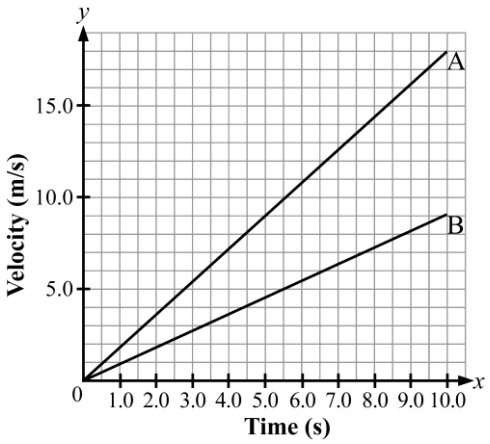
b) the displacement by the object in 10.0 s

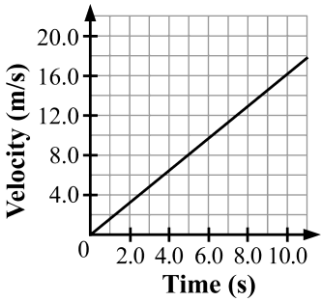
4. The following graph represents the motion of two objects A and B travelling right.



a) At 5.0 s, how many times faster is object A travelling than object B?

b) At 5.0 s, how much more distance did object A travel than object B?

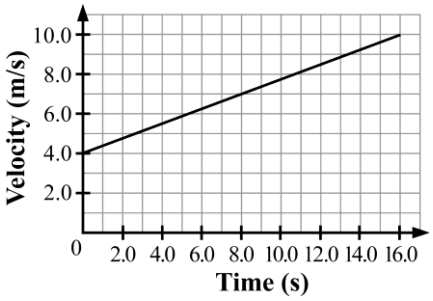
5. Given the following velocity-time graph for an object moving left, find



a) the acceleration of the object

b) the displacement by the object in 10.0 s

6. Given the following velocity-time graph for an object moving left, find



a) the acceleration of the object

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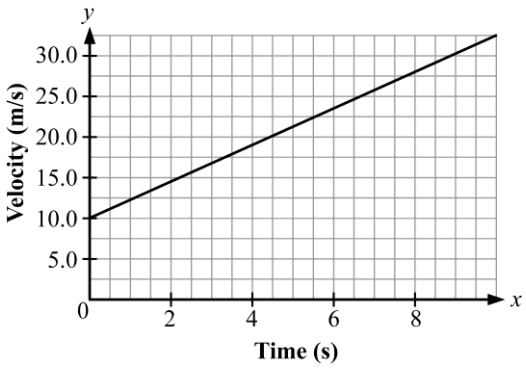
b) the displacement by the object in 16.0 s

7. Given the following velocity-time graph for an object moving right, find



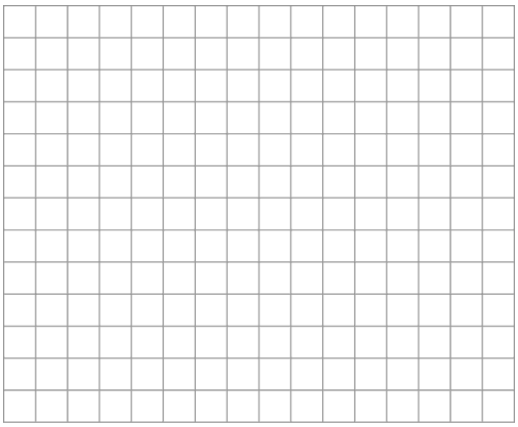
a) the acceleration of the object

b) the displacement by the object in 10.0 s

8. The data below shows the velocity of an object moving right at 1.0 s intervals.

Time (s)	Velocity (m/s)
0	12.0
1.0	15.3
2.0	18.6
3.0	21.9
4.0	25.2
5.0	28.5
6.0	31.8

a) Draw a velocity-time graph



b) Use the graph you drew to find

i) the acceleration of the object

ii) the displacement of the object during the 6.0 s described in the data table

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9. The dots below represent the position of an object every 0.10 s as it moves right along a horizontal line.

$t = 0$

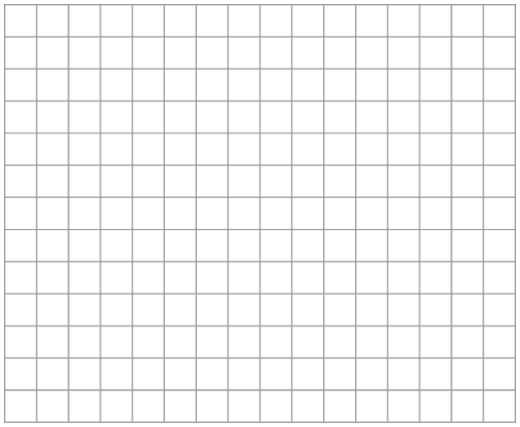
•• • • • • •

Analyze this motion by

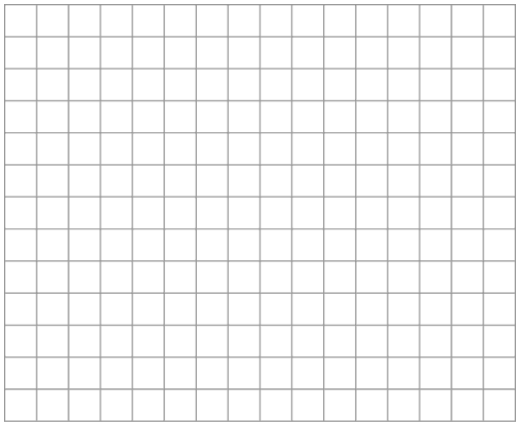
a) completing the following data table

Time (s)	Displacement from $t = 0$ (m)	Displacement during time interval (m)	Average velocity during time interval (m/s)
0			
0.10			
0.20			
0.30			
0.40			
0.50			
0.60			

b) drawing a position-time graph



c) drawing a velocity-time graph



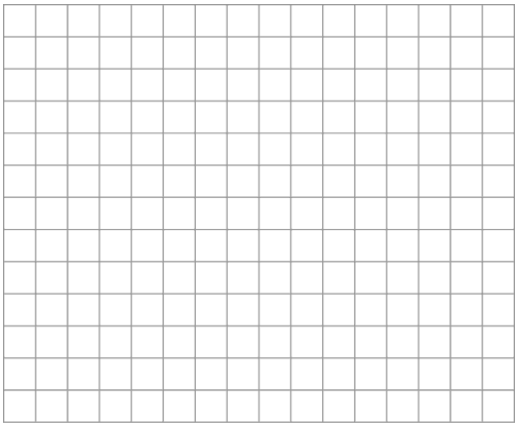
d) finding the acceleration of the object using your velocity-time graph

10. Given the following data obtained by moving an object to the right at a constant acceleration.

Time (s)	Displacement from $t = 0$ (m)	Displacement during time interval (m)	Average velocity during time interval (m/s)
0	0.0		
0.10	0.02		
0.20	0.09		
0.30	0.20		
0.40	0.36		
0.50	0.56		
0.60	0.80		
0.70	1.09		

a) complete the data table

b) draw a position-time graph



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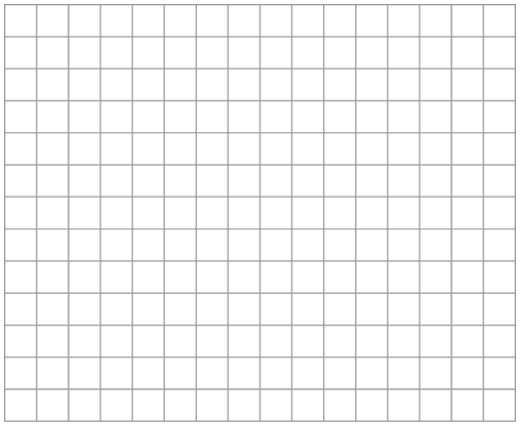
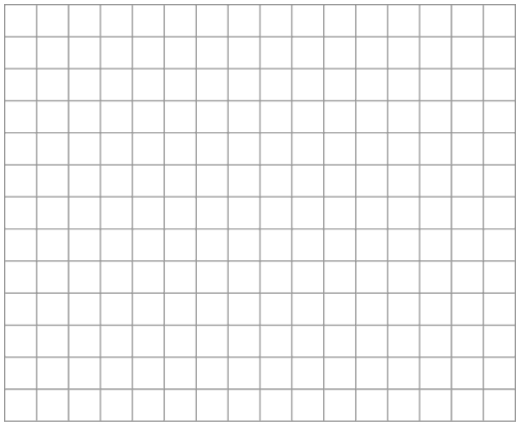
c) draw a velocity-time graph



d) draw an acceleration-time graph



e) find the velocity at 0.30 s and 0.60 s using your velocity-time graph

11. An object accelerates uniformly from rest to a velocity of 12.0 m/s left in 3.40 s. Calculate the acceleration of the object.

12. An object accelerates uniformly from rest. If the final velocity of the object after 4.7 s is 15 m/s right, what is the displacement?

13. An object accelerates uniformly from rest at a rate of 1.9 m/s^2 right for 5.0 s. Find

a) the displacement

b) the final velocity

c) the distance travelled

d) the final speed

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14. An object initially travelling at a velocity of 2.0 m/s left accelerates uniformly at a rate of 1.3 m/s^2 left. During this time of acceleration, the displacement of the object is 15 m left. Find

a) the final velocity

b) the final speed

15. An object accelerates uniformly from a velocity of 5.0 m/s right at a rate of 3.0 m/s^2 right. What is the velocity of the object at 2.9 s?

16. An object accelerates uniformly from rest. If it travels 26.0 m left, and reaches a velocity of 11.0 m/s left, how long was the object accelerating?

17. An object accelerates uniformly from rest for 8.10 s. If in this time the displacement of the object is 20.0 m to the right, what is the acceleration?

18. An object uniformly accelerates from 15 km/h right to 65 km/h right. If the rate of acceleration is 4.0 m/s^2, what is the displacement of the object during acceleration?

19. An object uniformly accelerates from rest and reaches a velocity of 122 km/h right in 10.5 s. What was the average velocity of the object?

20. An object accelerates uniformly from rest at a rate of 2.40 m/s^2 left. How many seconds was the object travelling when a velocity of 12.0 m/s left was reached?

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21. While accelerating uniformly from rest, an object is displaced 19.0 m right in 7.10 s. What is the velocity at this time?

22. An object travels 8.0 m left in 3.2 s while uniformly accelerating at a rate of 0.71 m/s^2 left. What was the initial velocity of the object?

23. An object uniformly accelerates from 15.0 m/s left to 35.0 m/s left. What is the rate of acceleration if the displacement during this time was 43.0 m left?

24. An object uniformly accelerates from a velocity of 7.0 m/s left to 19.0 m/s left. What is the average velocity of the object?

25. An object accelerates uniformly at a rate of 1.50 m/s^2 right for 10.0 s. If the velocity of the object reaches 25.0 m/s right at this time, what was the initial velocity?

26. An object is displaced 25.0 m right while accelerating uniformly. If a velocity of 14.0 m/s right is reached in 1.90 s, what was the initial velocity?

27. An object accelerates uniformly from rest for 5.6 s. What was the velocity of the object in this time if the displacement was 31.0 m to the left?

28. An object accelerates uniformly at a rate of 0.900 m/s^2 left. This object reaches a velocity of 25.0 m/s left while its displacement was 37.0 m left. What was the initial velocity of the object?

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29. A ball, starting from rest, accelerates uniformly down an incline at a rate of 1.4 m/s^2. Find

a) the distance travelled by the ball in 5.0 s

b) the velocity of the ball at 5.0 s

30. An object was travelling at an average velocity of 9.60 m/s to the right. If the time of travel was 2.70 s, what was the displacement of the object?

31. An object was uniformly accelerated from rest at a rate of 3.10 m/s^2 left. If this object reaches a velocity of 12.4 m/s left in 4.00 s, what is the displacement of the object while accelerating?

32. An object initially at rest is uniformly accelerated to a velocity of 10.0 m/s left in 2.50 s. If during this time the displacement of the object was 12.5 m left, what was the rate of acceleration?

33. An object was displaced 19.6 m right while uniformly accelerating from rest. If the average velocity of the object was 5.00 m/s right, what was the rate of acceleration?

34. An object uniformly accelerates at a rate of 1.00 m/s^2 right. While accelerating at this rate, the object is displaced 417.2 m right in a time of 27.0 s. What velocity did this object reach in this time?

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Lesson 4 FREELY FALLING OBJECTS

If you drop your pen it will fall to the floor, how is that motion described? Does it fall to the floor with uniform motion (uniform velocity), or uniform accelerated motion? Aristotle, the early Greek natural philosopher, came to the conclusion that objects fall with a constant velocity and that this velocity is proportional to the mass of the falling object. Galileo came to the conclusion that falling objects accelerate uniformly at the same rate.

NOTES

Galileo came to the conclusion that all objects accelerate uniformly at the same rate when they fall.

Questions

1. Why do you think Aristotle came to the conclusion that he did?

2. How did Galileo come to this conclusion about falling objects?

In 1971, Astronaut Scott dropped a feather and a hammer at the same time from the same height above the moon's surface. Which object (feather or hammer) hit the moon's surface first?

The acceleration of a freely falling object is called the acceleration due to gravity (*g*). Near the Earth's surface the magnitude of this acceleration is approximately 9.81 m/s^2. It should be noted that it varies slightly due to altitude and latitude. However, 9.81 m/s^2 will be used in problem solving unless stated otherwise.

$\vec{g} = 9.81\,\text{m/s}^2$ down

Problems involving freely falling objects are some of the best examples of uniform accelerated linear motion in one direction. Therefore, the problems that follow will be solved using the kinematic equations:

$$\vec{a} = \frac{\vec{v}_\text{f} - \vec{v}_\text{i}}{t}$$

$$\vec{d} = \left(\frac{\vec{v}_\text{f} + \vec{v}_\text{i}}{2}\right)t$$

$$\vec{d} = \vec{v}_\text{i}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v}_\text{f}^{\,2} = \vec{v}_\text{i}^{\,2} + 2\vec{a}\vec{d}$$

The acceleration due to gravity near the Earth's surface is approximately 9.81 m/s2.

NOTE: You may replace $\vec{a}$ in these equations with $\vec{g}$ if the acceleration is solely due to gravity.

Remember that the initial and final velocity of an object may also be denoted as $\vec{v}_0$ and $\vec{v}$, respectively.

NOTES

Example

A large steel ball is dropped from a height of 7.00 m above the floor. What is the velocity at which the object will strike the floor?

Solution

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
0	?	–9.81 m/s^2	–7.00 m	X

$$\vec{v}_f^{\,2} = \vec{v}_i^{\,2} + 2\vec{a}\vec{d}$$
$$= 2\left(-9.81\ \text{m/s}^2\right)\left(-7.00\ \text{m}\right)$$
$$= 137\ \text{m}^2/\text{s}^2$$
$$\vec{v}_f = -11.7\ \text{m/s or } 11.7\ \text{m/s down}$$

Acceleration due to gravity causes motion toward Earth's surface.

Example

A cement block falls from the roof of a building. If the time of fall was 5.60 s, what is the height of the building?

Solution

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
0	X	–9.81 m/s^2	?	5.60 s

The height of the building will equal the distance the object travels.

$$\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$
$$= \frac{1}{2}(-9.81\ \text{m/s}^2)(5.60\ \text{s})^2$$
$$= -154\ \text{m}$$

The negative in the answer is because the block travelled in the direction of the gravitational force, which is negative by convention. Therefore, the height of the building is 154 m.

Example

If you drop your pen from a height of 2.50 m above the floor, how long will it take to fall?

Solution

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
0	X	–9.81 m/s^2	–2.50 m	?

$$\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$
$$t = \sqrt{\frac{2\vec{d}}{\vec{g}}}$$
$$= \sqrt{\frac{2\left(-2.50\ \text{m}\right)}{-9.81\ \text{m/s}^2}}$$
$$= 0.714\ \text{s}$$

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PRACTICE EXERCISES

Formulas: $\vec{a} = \dfrac{\vec{v}_f - \vec{v}_i}{t}$ $\vec{d} = \left(\dfrac{\vec{v}_f + \vec{v}_i}{2}\right)t$ $\vec{d} = \vec{v}_i t + \dfrac{1}{2}\vec{a}t^2$ $\vec{v}_f^{\,2} = \vec{v}_i^{\,2} + 2\vec{a}\vec{d}$

1. A steel ball falls from a height of 15.0 m above the ground. How fast will it be travelling when it strikes the ground?

2. An apple falls from a tree, if the time of fall was 0.50 s, from what height did the apple fall?

3. A book falls from a shelf that is 1.75 m above the floor. How long will it take the book to reach the floor?

4. If you drop a coin from a height of 9.50 m above the ground, how fast will it be travelling when it reaches the ground?

5. An object is dropped from the roof of a building. If the object takes 2.5 s to reach the ground, what was the velocity of the object when it reached the ground?

6. A rock was thrown vertically downward from a bridge over a river. If the rock was released when it was 11.2 m above the water, and it took 0.550 s for the rock to reach the water, what was the velocity of the rock when it was released?

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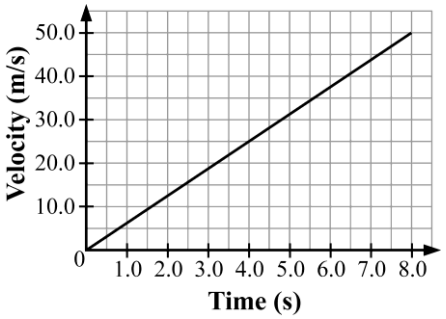
7. An egg is thrown vertically downward from a window. If the egg was released with a velocity of 10.0 m/s and it strikes the ground at a velocity of 25.0 m/s, how long did it take the egg to reach the ground?

8. A rock was thrown vertically downward. If the rock was released with a velocity of 5.0 m/s down and it hit the ground below at a velocity of 15.0 m/s, from what height was the rock released?

9. An object is thrown vertically downward. If the object hit the ground with a velocity of 10.0 m/s and it fell for 0.880 s, at what velocity was the object released?

10. A steel ball is dropped from a height of 50.0 m. How far does this ball travel during the 3rd second of fall?

11. The following velocity-time graph represents the motion of an object that accelerates uniformly to the left.



a) What is the average velocity during the 8.0 s of travel?

b) At what time does the object have an instantaneous velocity equal to the average velocity?

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12. a) A rock is dropped from a height of 7.0 m. What is the average velocity of the rock during the fall?

b) How long does it take the rock to fall?

c) At what time is the rock initially travelling at the average velocity?

13. You will note from questions 11 and 12 that the average velocity occurs at the mid-time of the motion. Using this, determine the time of fall when an object is dropped if the average velocity of the falling object was 12.0 m/s down.

14. When an object is dropped from a height of 10.0 m above the surface of Planet X, it takes 1.20 s for the object to reach the surface. What is the acceleration of a falling object near the surface of this planet?

15. When an object is dropped from a height of 24.0 m above the surface of Planet Y, it hits the surface at a velocity of 19.6 m/s. What is the acceleration of a falling object near the surface of this planet?

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16. When an object is dropped near the surface of Planet F, it reached a velocity of 11.0 m/s in 1.50 s. What is the acceleration of a falling object near the surface of Planet F?

17. A rock that is thrown vertically downward travels 10.0 m before hitting the ground. If it strikes the ground at a velocity of 15.0 m/s, how long did it take to reach the ground?

18. A rock that is thrown vertically downward takes 0.85 s to reach the ground. If it travels 9.2 m before reaching the ground, what is its velocity when it strikes the ground?

19. An object is dropped from a short distance above Planet T. If the average velocity during a fall of 40.0 m is 30.0 m/s, what is the acceleration of a falling object near the surface of Planet T?

20. When an object is dropped from a height of 20.0 m above the surface of Planet Z, it will fall 5.00 m during the 2nd second of the fall. What is the acceleration of a falling object near the surface of this planet?

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Lesson 5 HORIZONTAL UNIFORMLY ACCELERATED MOTION IN TWO DIRECTIONS ALONG A STRAIGHT LINE

NOTES

To this point, the direction of the vector quantities has always been in one direction along a line. Now, problems in which the vector quantities are in both directions along a straight line will be considered.

Example

A ball starts up an incline with a certain velocity. The ball slows down, comes to a stop, and starts to roll back down.

In this example:

$\vec{v}_i$ is up the incline

$\vec{a}$ is down the incline

$\vec{v}_f$ is down the incline

$\vec{d}$ is up the incline

(NOTE: Displacement becomes zero when the ball returns to its starting point and will be down if it rolls beyond its starting point in the opposite direction.)

PROBLEM SOLVING

In solving problems involving vectors in both directions along a straight line, continue to use the kinematic equations:

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$

$$\vec{d} = \left(\frac{\vec{v}_f + \vec{v}_i}{2} \right) t$$

$$\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v}_f^{\,2} = \vec{v}_i^{\,2} + 2\vec{a}\vec{d}$$

The direction of the motion will be indicated using positive (+) and negative (–) signs. It is customary to identify a direction to the right or up as positive (+) and a direction to the left or down as negative (–).

Direction along a line is indicated using a positive (+) and negative (–) sign.

SUMMARY

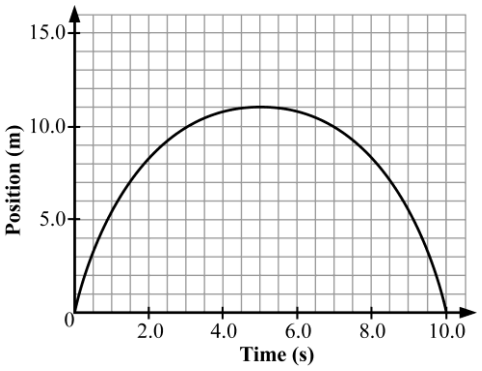
- The slope of a position-time graph is the velocity
- The slope of a velocity-time graph is the acceleration
- The area under a velocity-time graph is the displacement

NOTE:

- Slopes may be positive or negative
- Areas may be positive or negative
- The positive and negative nature of these quantities is due to the fact that the motion takes place on a line (two possible directions exist).

PRACTICE EXERCISES

1. The following position-time graph represents the motion of a steel ball rolling up an incline, coming to a stop, and returning back to its original position.



Find the velocity at

a) 3.0 s

b) 7.0 s

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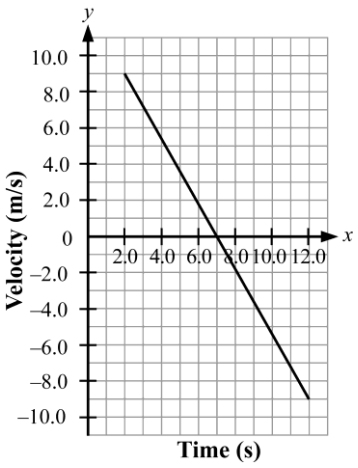
2. Given the following velocity-time graph for an object moving right along a line,

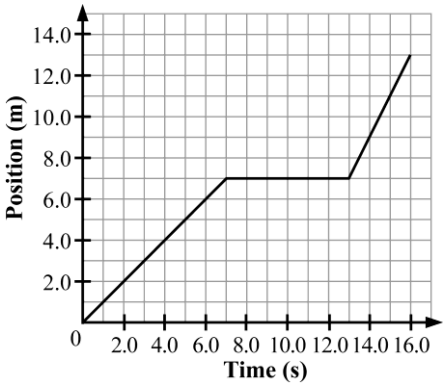


find the acceleration of the moving object at

a) 4.0 s

b) 10.0 s

3. Given the following position-time graph for an object moving right along a line,



find the

a) displacement of the object at 16.0 s

b) velocity at 5.0 s

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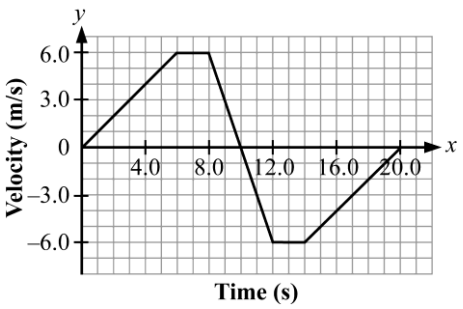
c) velocity at 9.0 s

d) velocity at 15.0 s

e) average velocity of the motion described

f) acceleration between 2.0 s and 6.0 s

4. The following is a velocity-time graph for an object moving along a line.



If the initial direction of the object is to the right, find the

a) velocity at 4.0 s

b) velocity at 7.0 s

c) velocity at 11.0 s

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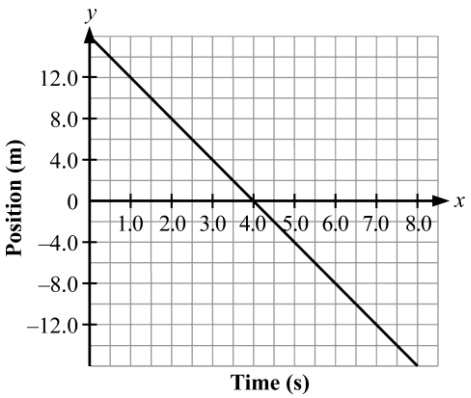
d) acceleration at 10.0 s

e) acceleration at 17.0 s

f) displacement at 10.0 s

g) average velocity during the total motion described

5. The following is a position-time graph for an object moving along a line.



If positive values are directed right, find the

a) velocity at 2.0 s

b) velocity at 8.0 s

c) acceleration between 2.0 s and 8.0 s

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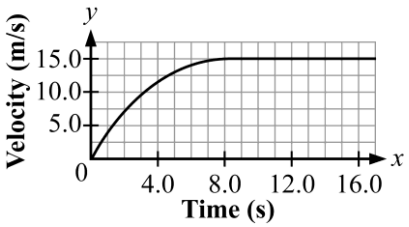
6. The following is a velocity-time graph for an object moving along a line.



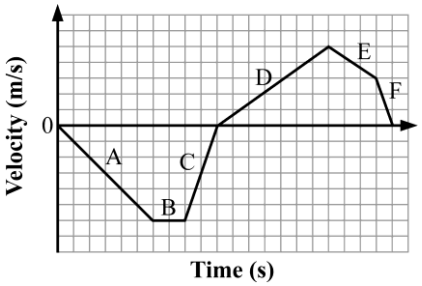
If the direction of the object is to the right, find the

a) velocity at 7.0 s

b) acceleration at 9.0 s

c) displacement between 8.0 s and 18.0 s

7. Given the following velocity-time graph for an object moving along a line,



in which section is the

a) displacement greatest (magnitude)?

b) speed the greatest?

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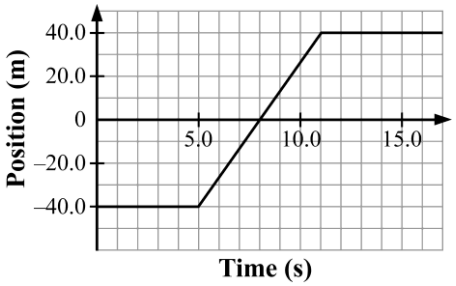
8. Given the following position-time graph for an object moving right along a line,



find the

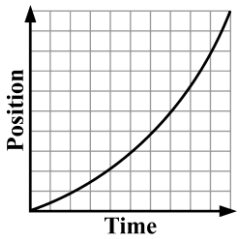
a) velocity at 8.0 s

b) velocity at 15.0 s

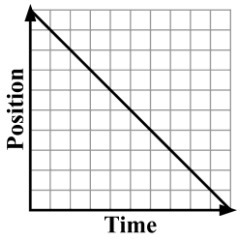
c) displacement between 4.0 s and 15.0 s

9. Given the following position-time graphs,

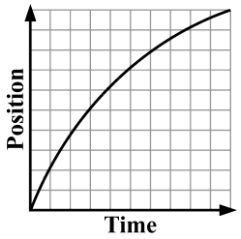
A.



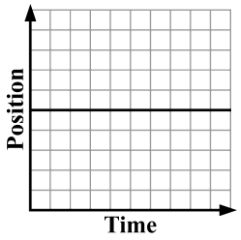
B.



C.



D.



which of these graphs represents

a) zero velocity?

b) motion in which the velocity is increasing?

c) motion in which the velocity is decreasing?

d) motion in which the velocity is constant?

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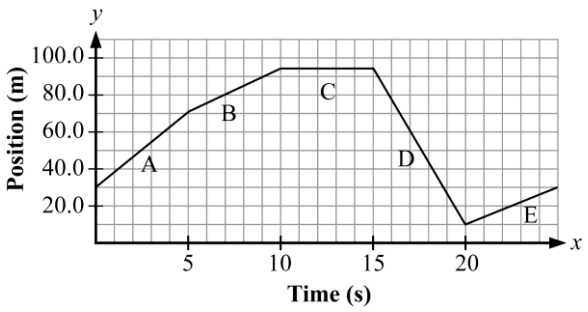
10. The following is a position-time graph for an object moving along a line.



If the initial direction of the motion is right,

a) find

i) the velocity of the object at 17.0 s

ii) the total distance travelled by the object from the beginning to the end of the motion described in the graph

iii) the displacement of the object from the beginning to the end of the motion described in the graph

iv) the average speed of the object during the motion described in the graph

b) During which time interval

i) does the object have a negative velocity?

ii) does the object reach its highest speed?

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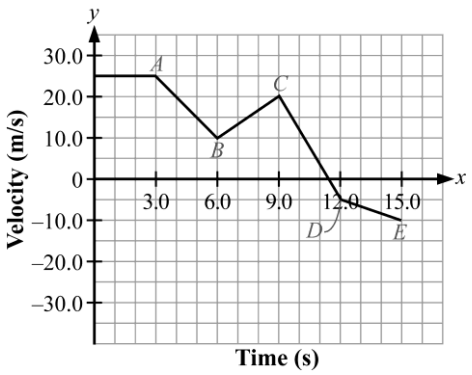
11. Given the following velocity-time graph for an object moving along a line



a) During which time interval is the object's velocity the greatest?

b) During which time interval is the acceleration of the object zero?

c) During which time interval is the magnitude of the acceleration of the object greatest?

12. The dots below represent the position of an object every 0.10 s as it moves up an incline.

The dots below represent the position of the same object as it continues its motion back down the incline.

Analyze this motion by

a) Completing the following data table

Time (s)	Displacement (m)	Displacement during time interval (m)	Average velocity during time interval (m/s)
0	0		
0.10	0.030		
0.20	0.055		
0.30	0.076		
0.40	0.094		
0.50	0.107		
0.60	0.115		
0.70	0.119		
0.80	0.120		
0.90	0.117		
1.00	0.109		
1.10	0.096		
1.20	0.079		
1.30	0.059		
1.40	0.034		

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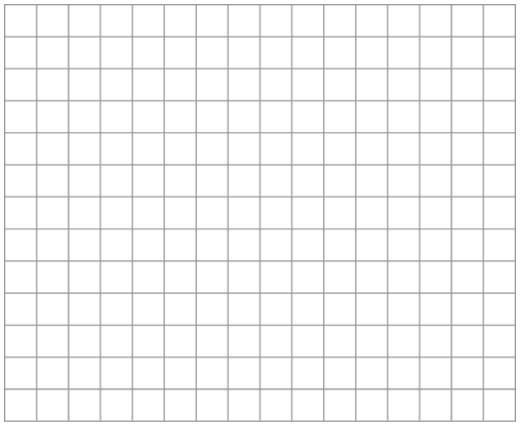
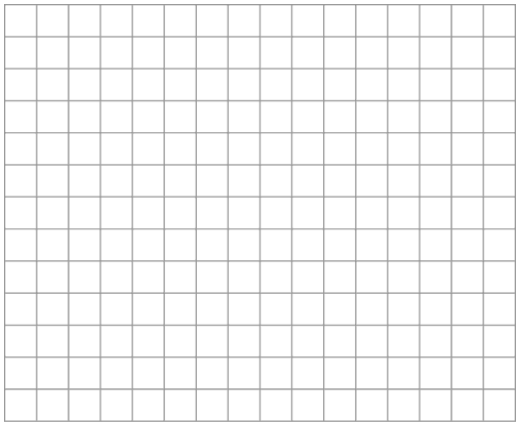
b) Drawing a position time graph



c) Drawing a velocity-time graph



d) finding

i) the velocity of the object using your position-time graph at

- 0.40 s

- 1.10 s

ii) the displacement of the object using your velocity-time graph at 1.40 s

iii) the acceleration of the object using your velocity-time graph

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Lesson 6 VERTICAL UNIFORMLY ACCELERATED MOTION IN TWO DIRECTIONS ALONG A STRAIGHT LINE

NOTES

When working with the mathematical analysis of uniformly accelerated motion along a straight line, it is important to keep in mind that:

- The direction up is positive (+).
- The direction down is negative (–).

Example

A stone is thrown vertically upward with an initial velocity of 25.2 m/s. Calculate the maximum displacement (height) of this stone.

Solution

$\vec{v}_0$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
25.2 m/s	0	$-9.81\ \text{m/s}^2$	?	X

Initial and final velocity may also be denoted by $\vec{v}_i$ and $\vec{v}$, respectively.

$$\vec{v}_f^{\,2} = \vec{v}_0^{\,2} + 2\vec{a}\vec{d}$$
$$0 = (25.2\ \text{m/s})^2 + 2(-9.81\ \text{m/s}^2)\vec{d}$$
$$\vec{d} = 32.4\ \text{m}$$

Example

A stone is dropped from a height of 32.4 m above the ground. Calculate the velocity of this object when it reaches the ground.

Solution

$\vec{v}_0$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
0	?	$-9.81\ \text{m/s}^2$	–32.4 m	X

$$\vec{v}_f^{\,2} = \vec{v}_0^{\,2} + 2\vec{a}\vec{d}$$
$$= 2(-9.81\ \text{m/s}^2)(-32.4\ \text{m})$$
$$\vec{v}_f = \sqrt{2(-9.81\ \text{m/s}^2)(-32.4\ \text{m})}$$
$$= -25.2\ \text{m/s or } 25.2\ \text{m/s down}$$

NOTE: The initial velocity in Example 1 and the final velocity in Example 2 are equal in magnitude. This will always be true when the same displacement up and down is involved (in the absence of friction).

NOTES

Example

A stone is thrown vertically upward with an initial velocity of 11 m/s. Calculate the time the stone is in the air.

Solution

$\vec{v}_0$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
11 m/s	–11 m/s	–9.81 m/s^2	X	?

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_0}{t}$$

$$t = \frac{\vec{v}_f - \vec{v}_0}{\vec{a}}$$

$$= \frac{-11 \text{ m/s} - 11 \text{ m/s}}{-9.81 \text{ m/s}^2}$$

$$= 2.2 \text{ s}$$

NOTE: Although the velocity in which the stone returned is not given, when an object is thrown vertically upward, it will return with the same speed but opposite direction.

Example

A ball is rolled up a constant slope with an initial velocity of 12.0 m/s. If the ball's displacement is 0.500 m up the slope after 3.60 s, what is the velocity of the ball at this time?

Solution

$\vec{v}_0$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
12.0 m/s	?	X	0.500 m	3.60 s

$$\vec{d} = \left(\frac{\vec{v}_f + \vec{v}_0}{2} \right) t$$

$$0.500 \text{ m} = \left(\frac{\vec{v}_f + 12.0 \text{ m/s}}{2} \right) (3.60 \text{ s})$$

$$\vec{v}_f = \frac{2(0.500 \text{ m})}{3.60 \text{ s}} - 12.0 \text{ m/s}$$

$$= -11.7 \text{ m/s or } 11.7 \text{ m/s down}$$

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PRACTICE EXERCISES

Formulas: $\vec{a} = \dfrac{\vec{v}_f - \vec{v}_0}{t}$ $\vec{d} = \left(\dfrac{\vec{v}_f + \vec{v}_0}{2}\right)t$ $\vec{d} = \vec{v}_0 t + \dfrac{1}{2}\vec{a}t^2$

$\vec{v}_f^{\,2} = \vec{v}_0^{\,2} + 2\vec{a}\vec{d}$

1. An object is thrown vertically upward with an initial velocity of 14.0 m/s. What is the displacement after 1.80 s?

2. A ball is rolled up a constant slope with an initial velocity of 9.3 m/s. What is the acceleration of the ball if its displacement is 1.9 m up the slope after 2.7 s?

3. A ball is rolled up a constant slope with an initial velocity of 11.0 m/s. After 9.3 s, the ball is rolling down the slope with a velocity of 7.3 m/s. What is the rate of acceleration of the ball on the slope?

4. A ball is rolled up a constant slope with an initial velocity of 9.4 m/s. After 3.0 s the ball is rolling down the slope with a velocity of 7.4 m/s. How far up the slope is the ball at this time?

5. An object is thrown vertically upward with an initial velocity of 15.0 m/s. How high is the object when it is travelling down with a velocity of 8.0 m/s?

6. An object is thrown vertically downward toward the ground with an initial velocity of 5.0 m/s. This object hit the ground with a velocity of 12.0 m/s. How long did the object take to reach the ground?

7. A ball is rolled up a constant slope. After 3.6 s it reaches its maximum displacement of 2.6 m, and then begins to roll back down. What was the initial velocity of the ball when it started up the slope?

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8. An object is thrown vertically upward with an initial velocity of 10.0 m/s. What is the velocity of the object when it is on its way back down and is 5.0 m above the point of release?

9. An object is thrown vertically upward with a velocity of 25.0 m/s. What is the velocity of this object after 3.0 s?

10. A ball is rolled up a constant slope with an initial velocity of 2.0 m/s. After 1.5 s, the ball is 2.8 m up the incline from the point of release. What is the velocity of the ball at this time?

11. An object is thrown vertically upward. If after 3.0 s the object has a vertical displacement up of 5.0 m, what was the initial velocity?

12. A ball is rolled up a constant slope with an initial velocity of 2.2 m/s. After 2.0 s, the ball is rolling down the slope with a velocity of 1.1 m/s. What is the acceleration of the ball?

13. A ball is rolled up a constant slope with an initial velocity of 2.5 m/s. When the displacement of the ball is 1.0 m up the slope it has a velocity of 1.6 m/s down. What is the acceleration of the ball?

14. A ball rolls 2.7 m up a constant slope before it comes to a stop. It then rolls back down. If the initial velocity of the ball was 2.0 m/s, how long does it take the ball to return to its original position?

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15. While on Planet X, an object is thrown vertically upward with an initial velocity of 5.0 m/s. If this object returns to the point of release in 3.0 s, what is the acceleration of a freely falling object on this planet?

16. An object is thrown vertically upward from a helicopter that is hovering 30.0 m above the ground. The initial velocity of the object was 20.0 m/s.

a) Calculate the velocity with which the object hits the ground.

b) Calculate the time it took for the object to reach the ground.

17. While on Planet X, an object is thrown vertically upward with an initial velocity of 5.0 m/s. If this object returns to the point of release in 3.0 s, what is the acceleration of a freely falling object on this planet? Use a different method than in question 15.

18. While riding on an amusement park vehicle, you drop an object. The vehicle was rising vertically at a velocity of 11.0 m/s and was 5.0 m above the ground when the object was dropped. How long does it take the object to reach the ground?

19. An object is thrown vertically upward. If this object takes 5.30 s to go up and down, what height did it reach?

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20. An object is dropped from a height of 25.0 m above the ground. What is the average velocity of the object as it falls to the ground?

21. An object is thrown upward from the ground with an initial velocity of 14.0 m/s. When the object reaches its maximum displacement (height) find

a) its displacement from the ground

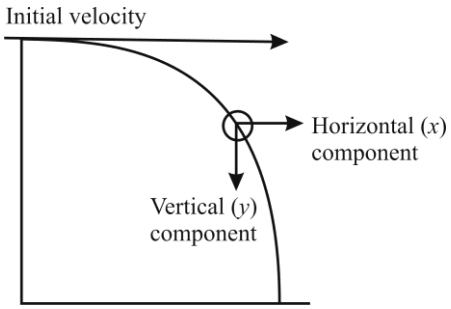
b) its time of travel

22. An object is rolled up an incline. If the object is 2.75 m up the incline after 4.50 s and rolling back down at a velocity of 1.90 m/s, what is the acceleration of the object?

Lesson 7 PROJECTILE MOTION

NOTES

A baseball thrown by a baseball player, a football kicked by a football player, a bullet fired at a distant target, and an object dropped from a plane are all examples of projectile motion. When an object is thrown into the air, it is a projectile. In the previous sections, projectiles that were thrown vertically upward, vertically downward, and dropped from varying heights were discussed. In this section, projectiles that are thrown horizontally through the air will be examined. These projectiles also are affected vertically by acceleration due to gravity. Since the motion of projectiles is horizontal and vertical, projectile motion problems are in dealt with in two dimensions. That is, vertical and horizontal directions must be considered together.



When working in two dimensions, the vectors are broken up into horizontal and vertical components. The horizontal motion will always be uniform motion (constant velocity) and will be described by the equation:

$$\vec{v}_x = \frac{\vec{d}_x}{t}$$

The vertical motion is uniformly accelerated motion (constant acceleration) due to gravity. This motion will be described by the equations:

$$\vec{a}_y = \frac{\vec{v}_{yf} - \vec{v}_{y0}}{t}$$

$$\vec{d}_y = \left(\frac{\vec{v}_{yf} + \vec{v}_{y0}}{2} \right) t$$

$$\vec{d}_y = \vec{v}_{y0} t + \frac{1}{2} \vec{a}_y t^2$$

$$\left(\vec{v}_{yf} \right)^2 = \left(\vec{v}_{y0} \right)^2 + 2 \vec{a}_y \vec{d}_y$$

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NOTES

In two dimensions, the relation of displacement, velocity, and acceleration with their corresponding *x*- and *y*-components are:

$\vec{d} = \vec{d}_x + \vec{d}_y$

$\vec{v} = \vec{v}_x + \vec{v}_y$

$\vec{a} = \vec{a}_x + \vec{a}_y$

For the following projectile motion problems, the motion should always be broken up into its *x*- and *y*-components.

Example

An object is thrown horizontally at a velocity of 20.0 m/s from the top of a building 50.0 m tall. How far from the base of the building did the object hit the ground?

Solution

Find the horizontal component of displacement.

$\therefore$ use $\vec{v}_x = \dfrac{\vec{d}_x}{t}$ or $d_x = \vec{v}_x t$

Find t from the vertical component.

$\vec{v}_{y0}$	$\vec{v}_{yf}$	$\vec{a}_y$	$\vec{d}_y$	t
0	X	$-9.81\ \text{m/s}^2$	$-50.0\ \text{m}$	?

$$\vec{d}_y = \vec{v}_{y0} t + \frac{1}{2}\vec{a}_y t^2$$

$$-50.0\ \text{m} = \frac{1}{2}\left(-9.81\ \text{m/s}^2\right) t^2$$

$$t = \sqrt{\frac{2\left(-50.0\ \text{m}\right)}{-9.81\ \text{m/s}^2}}$$

$$= 3.19\ \text{s}$$

Now use *t* to find the horizontal component of displacement:

$$\vec{d}_x = \vec{v}_x t$$

$$= \left(20.0\ \text{m/s}\right)\left(3.19\ \text{s}\right)$$

$$= 63.8\ \text{m}$$

PRACTICE EXERCISES

1. An object is thrown horizontally at a velocity of 10.0 m/s from the top of a 90.0 m building. Calculate the distance from the base of the building that the object will hit the ground.

2. An object is thrown horizontally at a velocity of 25.0 m/s from the top of a 1.50×10^2 m building. Calculate the distance from the base of the building that the object will hit the ground.

3. An object is thrown horizontally at a velocity of 18.0 m/s from the top of a cliff. If the object hit the ground 100.0 m from the base of the cliff, how high is the cliff?

4. An object is thrown horizontally at a velocity of 20.0 m/s from the top of a cliff. If the object hit the ground 48.0 m from the base of the cliff, how high is the cliff?

5. An object is thrown horizontally from the top of a building at a velocity of 15.0 m/s. If the object takes 5.50 s to reach the ground, how high is the building?

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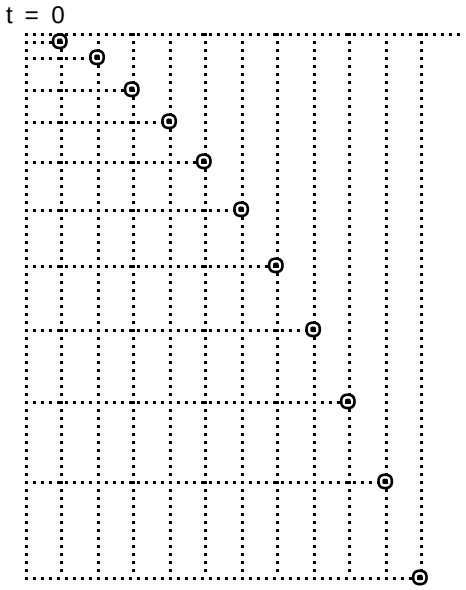
6. An object is thrown horizontally from the top of a cliff at a velocity of 20.0 m/s. If the object takes 4.20 s to reach the ground, how far from the base of the cliff did the object hit the ground?

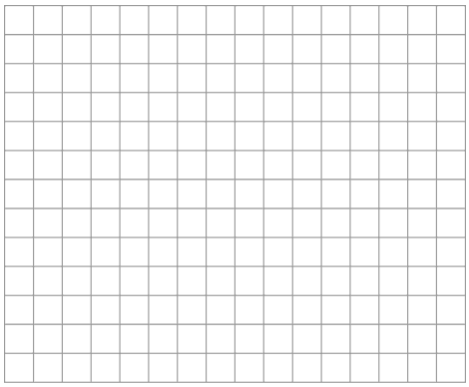
7. The dots below represent the position of a projectile every 0.10 s as it is projected horizontally to the right along an inclined air table. For this question, consider down and right as the positive directions.



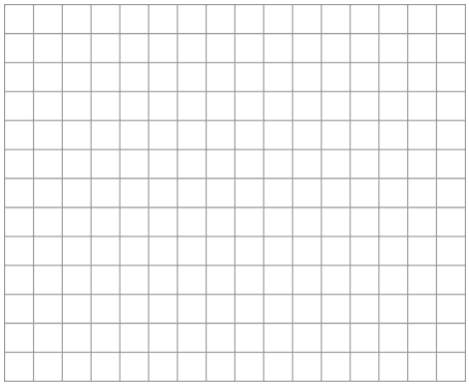
a) Analyze this motion by completing the following table:

Time (s)	Displacement from $t = 0$ ($\times 10^{-2}$ m)		Displacement during time interval ($\times 10^{-2}$ m)		Average velocity during time interval ($\times 10^{-2}$ m/s)	
	Horizontal	**Vertical**	**Horizontal**	**Vertical**	**Horizontal**	**Vertical**
0	0	0				
0.10	0.5	0.15				
0.20	1.0	0.30				
0.30	1.5	0.70				
0.40	2.0	1.1				
0.50	2.5	1.6				
0.60	3.0	2.2				
0.70	3.5	2.9				
0.80	4.0	3.7				
0.90	4.5	4.6				
1.00	5.0	5.6				
1.10	5.5	6.7				

b) Using your horizontal velocity, draw a velocity-time graph. For this question, consider down as the positive direction.



c) Using your vertical velocity, draw a velocity-time graph. For this question, consider down as the positive direction.



d) Using your graphs, find

i) the horizontal acceleration

ii) the vertical acceleration

8. An object is thrown horizontally from the top of an 85.0 m building. If the object hits the ground 67.8 m from the base of the building, what was the horizontal velocity of the object?

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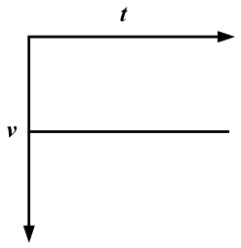
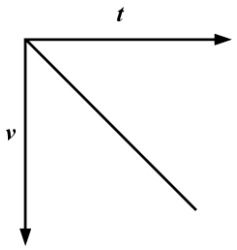
PRACTICE TEST

1. Which of the following graphs **best** represents a velocity-time graph for a falling object, initially at rest, in which air resistance can be ignored?

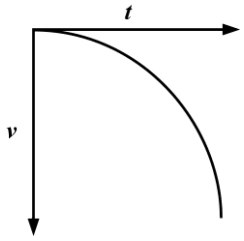
A.



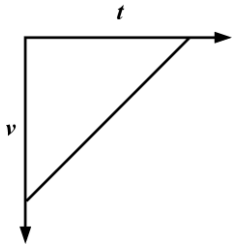
B.



C.

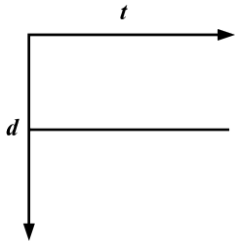


D.

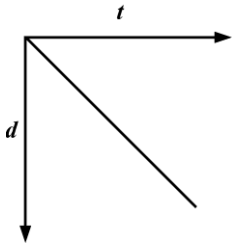


2. Which of the following graphs **best** represents a displacement-time graph for a falling object, initially at rest, in which air resistance can be ignored?

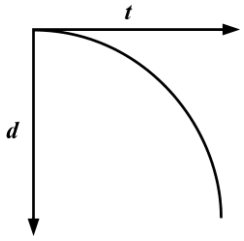
A.



B.



C.



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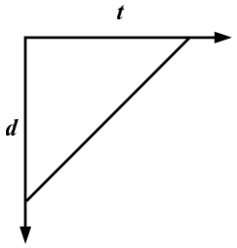
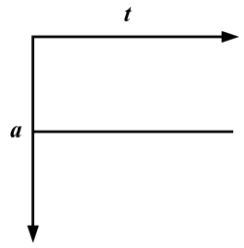
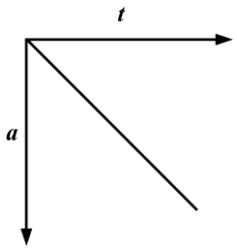


3. Which of the following graphs **best** represents an acceleration-time graph for a falling object in which air resistance can be ignored?

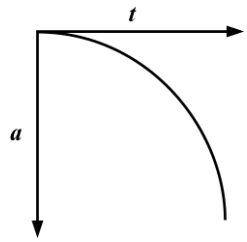
A.



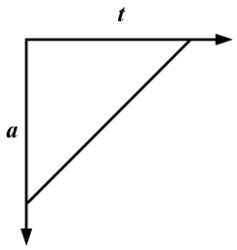
B.



C.

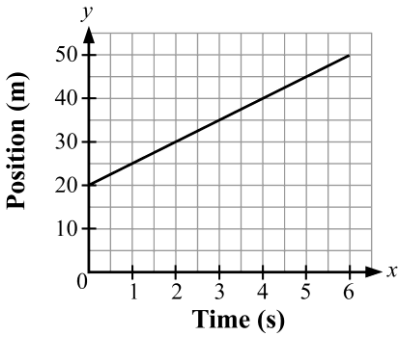


D.



4. Explain the difference between instantaneous and average speed.

5. Given the following position-time graph for an object travelling to the right along a straight horizontal line, what is the velocity of the object?



A. 5.0 m/s right

B. 8.3 m/s right

C. 0.12 m/s right

D. 0.20 m/s right

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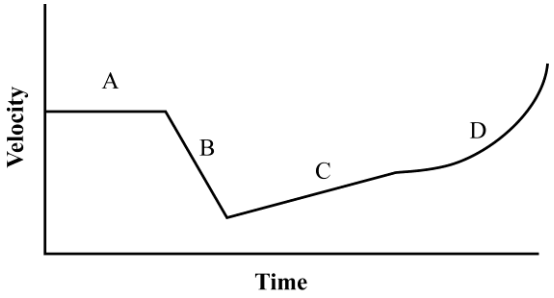
6. Describe how acceleration, velocity, and displacement change for an object undergoing uniform motion.

7. A ball player throws a ball straight up into the air and catches it a short time later. Describe the direction and magnitude of the velocity vector of the ball as it is released, at its maximum height, and right before it is caught.

8. The following is a velocity-time graph for an object undergoing free fall.



What is the displacement of the object during the six seconds of motion described in the graph?

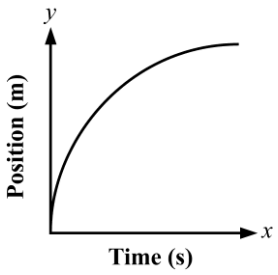
A. 5.0 m

B. 90 m

C. 180 m

D. 210 m

9. Describe the motion of an object in terms of velocity for the following graph.



10. A ball is thrown straight up into the air with a velocity of 8.0 m/s and is caught in the same spot it was thrown a short while later.

a) What velocity will it have when it is caught?

b) What is the acceleration of the object before it reaches its highest point?

c) What is the acceleration of the object at its highest point?

11. Describe the magnitude and direction of the displacement vector of a ball as it rolls a short distance up an incline and then a long distance back down.

12. A ball rolls 3.2 m up a constant slope before it comes to a stop. If the initial velocity of the ball was 2.2 m/s, how long (time) does it take the ball to roll up the slope?

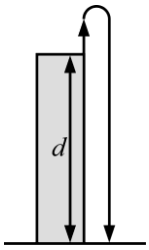
A. 8.6 s

B. 2.9 s

C. 4.2 s

D. 0.65 s

13. An object to thrown straight up into the air from the top of a building at a velocity of 15 m/s as shown in the diagram. If it hits the ground in 8.00 s, what is the height of the building?



A. 1.6×10^2 m

B. 1.9×10^2 m

C. 3.1×10^2m

D. 4.3×10^2 m

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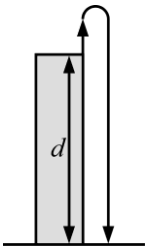
14. An object is thrown straight up into the air from the top of a building at a velocity of 12 m/s as shown in the diagram. If it hits the ground in 8.0 s, what is the speed of the object when it hits the ground?



A. 12 m/s

B. 66 m/s

C. 79 m/s

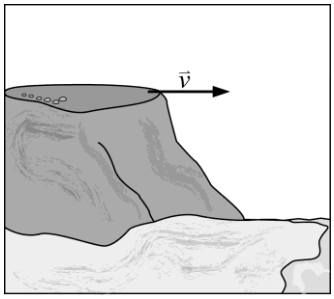
D. 91 m/s

15. Two objects are thrown by the same student at the same time. One is thrown with velocity of 5.0 m/s upward and the other is thrown with velocity of 5.0 m/s downward. Assume the effects of friction are negligible.

a) Which object hits the ground with a higher velocity?

b) Using principles of physics, explain the reasoning for your choice in part a).

16. Ignoring air friction, which of the following objects will reach the ground first if they are thrown horizontally from the top of a high cliff as shown in the diagram?



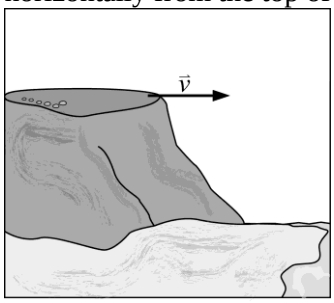
i) A 2.0 kg object is thrown at 10.0 m/s
ii) A 4.0 kg object is thrown at 10.0 m/s

A. The 2.0 kg object

B. The 4.0 kg object

C. They will both reach the ground at the same time.

17. Ignoring air friction, which of the following objects will reach the ground first if they are thrown horizontally from the top of a high cliff as shown in the diagram?



i) A 2.0 kg object is thrown at 10.0 m/s
ii) A 2.0 kg object is thrown at 15.0 m/s

A. The 2.0 kg object thrown at 10.0 m/s

B. The 2.0 kg object thrown at 15.0 m/s

C. They will both reach the ground at the same time.

18. A student stands at the edge of a cliff and throws a rock as far and high as he can. Ignoring friction, describe the rock's acceleration and velocity for both the horizontal and vertical components of the projectile motion.

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19. Ignoring air friction, which of the following objects will reach the ground first if they are thrown horizontally from the top of cliffs as described?

i) A 4.0 kg object is thrown from the top of a 75.0 m cliff at a velocity of 15.0 m/s
ii) A 2.0 kg object is thrown from the top of a 50.0 m cliff at a velocity of 12.0 m/s
iii) A 1.0 kg object is thrown from the top of a 25.0 m cliff at a velocity of 10.0 m/s

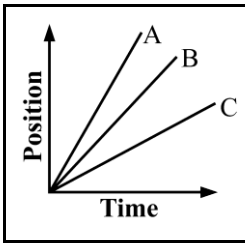
A. The 4.0 kg object

B. The 2.0 kg object

D. The 1.0 kg object

D. There is not enough information to determine the answer.

Use the following information to answer questions 20 and 21.



These position time graphs represent the motion of three objects.

20. Which object has the greatest velocity?

A. A

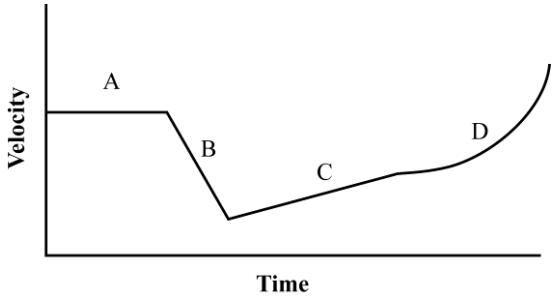
B. B

C. C

D. They all have the same velocity.

21. Which object has the greatest acceleration? How do you know this?

22. The following is a velocity-time graph of an object moving along a horizontal surface.

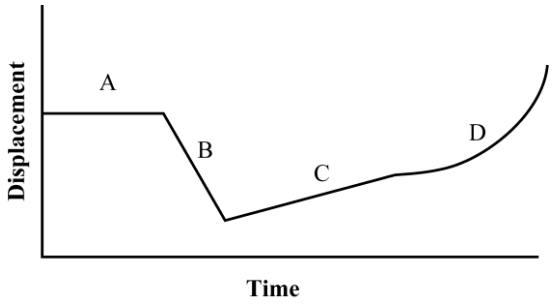


In which section does the object experience the greatest constant acceleration?

A. A **B.** B

C. C **D.** D

23. The following is a position-time graph of an object moving along a horizontal surface.



In which section does the object experience the greatest acceleration?

A. A **B.** B

C. C **D.** D

24. For an object that is thrown upward, which of the following statements is correct?

A. The acceleration of the object is greater as it travels up.

B. The acceleration of the object is greater as it travels down.

C. The acceleration of the object at the top of its motion is zero.

D. The acceleration of the object is constant as it travels up and down.

25. If an object is thrown horizontally on Earth's surface, does it fall to the ground due to a change in velocity or a change in acceleration? Explain.

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FORCES

BIG IDEA

Forces influence the motion of an object.

As you begin this unit, you should consider the following questions…

- How can forces change the motion of an object?
- How can Newton's laws be used to explain changes in motion?
- What are some factors that affect the force of friction?
- How do weight measurements change when measured in a stationary elevator compared to an accelerating elevator?
- How can free-body diagrams help develop equations that describe the motion of objects?
- What are the limitations of free-body diagrams?

Lesson	Page	Completed on
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2. Newton's Third Law of Motion	112	
3. Force due to Gravity	115	
4. Contact Forces	119	
5. Net Force and Motion	126	
Practice Test	151	
Answers and Solutions	at the back of the book	

When you are finished this unit, you are expected to know…

- The types of **contact forces, for example normal force, spring force, tension force, and frictional force, and the factors that affect magnitude and direction**
- The relationship between **mass, force of gravity, and apparent weight**.
- **Newton's laws of motion** (first, **the concept of mass as a measure of inertia**; second, **net force from one or more forces**; and third, **actions/reactions happen at the same time in pairs**) and the use of **free-body diagrams**.
- The impact of **balanced and unbalanced forces in systems**, including **one-body and multi-body systems**, **inclined planes**, **angled forces**, and **elevators**

Lesson 1 FORCES AND NEWTON'S LAWS OF MOTION

NOTES

Kinematics describes *how* an object moves; dynamics, which is the study of forces on an object, deals with *why* an object moves.

In 1665, when Isaac Newton was 23 years old, he formulated the three laws of motion. These laws help explain the forces that influence why an object moves or does not move.

FORCES

Force is any external push or pull on an object. A force will cause an object to accelerate, such as the throwing of a ball or the pushing of a car. The motion of these objects is the result of an unbalanced force acting upon them. However, forces do not always result in motion of an object because the force may be balanced by another equal and opposite force. For example, consider a student sitting motionless in her chair. The force of gravity is constantly pushing her toward the ground yet she remains motionless. This is because there is an equal and opposite force exerted on her by the chair that keeps her from moving. The forces acting on the student are balanced.

The unit of force is the newton (N).

F is the symbol for force.

Force, *F*, is a vector quantity and is measured in units of newtons (N). The concept of force can be further understood by examining Newton's laws of motion.

NEWTON'S FIRST LAW OF MOTION

Newton's first law:
An object will remain at constant velocity unless acted on by an unbalanced force.

Newton's first law states that an object will remain at constant velocity (including zero velocity) unless acted on by an unbalanced force. That is, if no forces act on an object, its acceleration is zero.

Newton's first law of motion is sometimes referred to as the law of inertia. Inertia is the tendency for an object to remain at constant velocity.

The symbol for net force is F_{net}.

Describing some familiar scenarios will help to illustrate this law. If you are in a car travelling at 50 km/h when it hits a concrete wall head on, the car will come to a sudden stop because of the extreme force acting on it. However, if you are not wearing your seat belt, you will continue at 50 km/h until forces acting on you bring you to a stop. Seatbelts are worn to keep people in the car to reduce the chance of severe injury.

If you are sitting at a stop light when someone runs into your car from behind at a velocity of 40 km/h, your car experiences a sudden force which causes it to increase its velocity. Your body will move too from the force of the seat. However, your head, which is somewhat loosely attached to your body, will have a certain tendency to remain stopped (at rest). So as your body is suddenly thrust forward, your head (remaining at rest) is caused to be snapped back. This motion results in whiplash injuries. Head rests in cars are designed to help reduce these injuries.

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NOTES

Imagine you are on a transit bus standing in the aisle travelling at 50 km/h when the bus driver brakes hard. What will happen to you? Imagine you are on this same bus travelling at 30 km/h and the bus suddenly turns sharply to the right. What happens to you?

These are illustrations of Newton's first law of motion (law of inertia). The inertia of an object is related to its mass.

MASS

Mass can be described as a measure of an object's tendency to resist changes in motion. The higher the mass of an object, the more it resists a change. That is, mass is a quantitative measure of an object's inertia. From your experiences, you know that the greater the mass of an object, the greater its inertia. Imagine kicking a soccer ball and then a ten pin bowling ball. You would notice that the bowling ball resisted a change in velocity to a much greater extent than did the soccer ball. Mass, *m*, is measured in units of kilograms (kg) and is a scalar quantity.

NEWTON'S SECOND LAW OF MOTION

Newton's first law describes what happens to an object when no forces act on it or when the net force is zero. The object will either remain stationary or continue to move at a constant velocity in a straight line. Newton's second law describes what happens to an object when a force acts on an object resulting in a non-zero unbalanced net force. Newton's second law of motion, the fundamental law of dynamics, states that the rate of change in an object's velocity (acceleration) is directly proportional to the net force and inversely proportional to the object's mass.

$$\vec{a} \propto \vec{F}_{\text{net}}$$

$$\vec{a} \propto \frac{1}{m}$$

This leads to:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$$

$$\vec{F}_{\text{net}} = m\vec{a}$$

An unbalanced force occurs when the sum of all forces (net force) acting on the object is non-zero. For multiple forces acting on an object, the net force can be described as:

$$\vec{F}_{\text{net}} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots$$

Net force is the sum of all forces acting on an object.

Symbol: F_{net}

Newton's second law: The rate of change in an object's velocity is directly proportional to the net force and inversely proportional to the object's mass.

Force is the product of mass times acceleration.

$N = kg \bullet m/s^2$

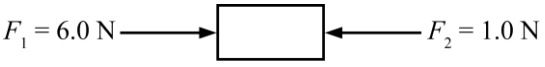
Inertia is the tendency for an object to remain at a constant velocity.

NOTES

To demonstrate the use of this equation, consider two equal and opposite applied forces acting on a motionless 2.0 kg object. Choosing right as the positive direction:

$$\begin{aligned}\vec{F}_{net} &= \vec{F}_1 + \vec{F}_2 \\ &= 6.0\text{ N} - 6.0\text{ N} \\ &= 0 \\ \vec{F}_{net} &= m\vec{a} \\ \vec{a} &= 0\end{aligned}$$

The net force on this object is zero. The object's acceleration remains zero and the forces acting on it are balanced. Therefore, the object remains motionless. Now consider two different forces acting on the same object:



$$\begin{aligned}\vec{F}_{net} &= \vec{F}_1 + \vec{F}_2 \\ &= 6.0\text{ N} - 1.0\text{ N} \\ &= 5.0\text{ N right} \\ \vec{F}_{net} &= m\vec{a} \\ \vec{a} &= \frac{5.0\text{ N}}{2.0\text{ kg}} \\ &= 2.5\text{ m/s}^2\text{ right}\end{aligned}$$

The net force on this object is non-zero. The object's acceleration is to the right since the net force is unbalanced and pushes with 5.0 N rightward. It is important to understand that force does not cause motion since an object can be moving in a straight line with a constant velocity with no forces acting on it. Force causes a *change* in motion of an object that is measured by the object's acceleration. Force is the product of mass times acceleration. Therefore, the unit of force (newton) can also be expressed as the product of mass units (kg) and acceleration units (m/s^2). The greater the mass of an object, the more force required to accelerate it.

Example

A net force of 30.0 N left acts on a 10.0 kg object. What is the acceleration of the object?

Solution

$$\begin{aligned}\vec{F}_{net} &= m\vec{a} \\ \vec{a} &= \frac{\vec{F}_{net}}{m} \\ &= \frac{30.0\text{ N}}{10.0\text{ kg}} \\ &= 3.00\text{ m/s}^2\text{ left}\end{aligned}$$

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NOTES

Example

A 22 kg object accelerates uniformly from rest to a velocity of 2.5 m/s to the left in 8.7 s. What is the net force acting on the car during this acceleration?

Solution

$$\begin{aligned}\vec{a} &= \frac{\vec{v}_f - \vec{v}_0}{t} \\ &= \frac{2.5 \text{ m/s} - 0}{8.7 \text{ s}} \\ &= 0.287 \text{ m/s}^2 \text{ left}\end{aligned}$$

$$\begin{aligned}\vec{F}_{net} &= m\vec{a} \\ &= (22 \text{ kg})(0.287 \text{ m/s}^2) \\ &= 6.3 \text{ N left}\end{aligned}$$

PRACTICE EXERCISES

Formula: $\vec{F}_{\text{net}} = m\vec{a}$

1. A net force of 9.0 N is used to push a 20.0 kg object to the right. What is the acceleration of the object?

2. A net force of 15.0 N is used to pull an object right. If the acceleration of this object is 8.0 m/s^2, what is the mass of the object?

3. A 16.0 kg object is accelerated at a rate of 2.0 m/s^2 to the left by a net force. What is the net force?

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4. A 12.0 kg object is accelerated by a net force of 10.2 N to the right. What is the acceleration of the object?

5. A 5.2 kg object is accelerating at a rate of 6.0 m/s^2 to the right. What is the magnitude of the net force acting on the object?

6. If an 18 kg object has a net force of 2.0 N pushing it to the left, what is the acceleration of the object?

7. A 925 kg car accelerates to the left uniformly from rest to a velocity of 25.0 m/s in 10.0 s. What is the net force acting on the car during this acceleration?

8. A 1.08×10^3 kg car uniformly accelerates to the right for 12.0 s from rest. During this time the car travels 132 m. What is the net force acting on the car during this acceleration?

9. A 1.20×10^3 kg car accelerates uniformly to the right from 5.0 m/s to 12 m/s. During this acceleration the car travels 94 m. What is the net force acting on the car during this acceleration?

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10. A net force of 2.5×10^3 N acts on an object for 5.0 s. During this time the object accelerates to the right from rest to a velocity of 48 km/h. What is the mass of this object?

11. A net force of 6.6 N acts on a 9.0 kg object. If this object accelerates uniformly from rest to a velocity of 3.0 m/s to the right,

a) How far did the object travel while accelerating?

b) What is the time of acceleration?

Lesson 2 NEWTON'S THIRD LAW OF MOTION

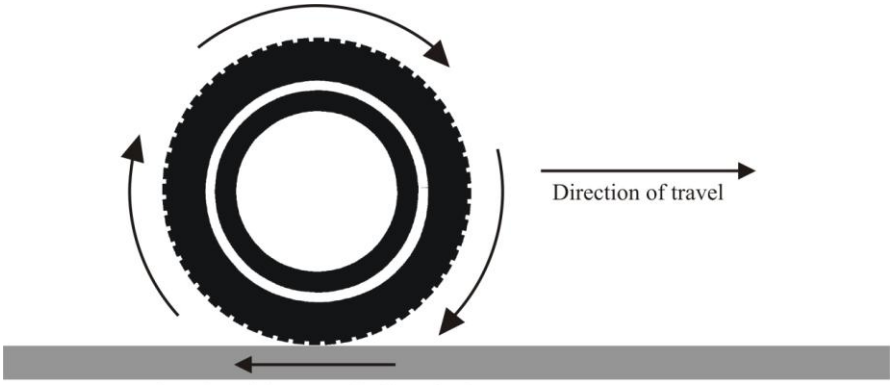
NOTES

Newton's third law: For every action (force) there is an equal but opposite reaction (force).

Newton's third law of motion states that when an object exerts a force on a second object, that the second object exerts an equal and opposite force on the first. Often, this law is stated in terms of action and reaction forces.

For every action (force) there is an equal but opposite reaction (force).
$\vec{F}_1 = -\vec{F}_2$

For example, if you push on your desk, the desk pushes back with an equal amount of force. Birds can fly because when they push down with their wings, the air pushes up and provides lift. In the image below, a car's tires exert force in the opposite direction to which the car travels.



The wheel exerts force upon the road while the road exerts an equal and opposite force upon the wheel. The force exerted by the wheel is not enough to move the road (obviously). The force exerted by the road upon the wheel is enough to propel the car down the road.

Example

While standing on a horizontal frictionless surface, two students push against each other. One student has a mass of 35 kg and the other 45 kg. If the acceleration of the 35 kg student is 0.75 m/s^2 to the right, what is the acceleration of the 45 kg student?

Solution

$$\vec{F}_1 = -\vec{F}_2$$
$$m_1\vec{a}_1 = -m_2\vec{a}_2$$
$$(35\text{ kg})(0.75\text{ m/s}^2) = -(45\text{ kg})\vec{a}_2$$
$$\vec{a}_2 = -0.58\text{ m/s}^2 \text{ or } 0.58\text{ m/s}^2 \text{ left}$$

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PRACTICE EXERCISES

Formula: $\vec{F}_1 = -\vec{F}_2$

1. While standing on a horizontal frictionless surface, two students, A and B, push against each other. Student A has a mass of 38 kg and during the push is accelerating right at a rate of 0.60 m/s^2. If student B is accelerating left during the push at a rate of 0.75 m/s^2, what is her mass?

2. While standing on a horizontal frictionless surface a 50.0 kg student pushes against a wall with an average force of 125 N to the right for 0.110 s. Calculate the velocity of this student at 0.110 s.

3. A 9.8×10^3 kg rocket is travels past you to your right along a horizontal frictionless rail at a velocity of 11 m/s. The rocket is then accelerated uniformly to a velocity of 22 m/s in a time of 0.75 s by the expulsion of hot gases. What is the average force at which the gases are expelled by the rocket?

4. While standing on a horizontal frictionless surface, a 45 kg student throws a 3.0 kg object to her right. During the throw the object was accelerated horizontally through a distance of 0.60 m from rest to a velocity of 9.6 m/s. Calculate the velocity of the student when the object is released.

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Lesson 3 FORCE DUE TO GRAVITY

NOTES

If you throw an object into the air, it will return to the Earth. If you drop an object from some distance from the floor, it will fall to the floor. Why is this?

Isaac Newton proposed that Earth exerts a force on objects near its surface. The story goes that Newton was sitting under an apple tree thinking about the motion of the moon around Earth when an apple broke free from its branch and fell to the ground. Newton decided that the force that caused the apple to fall to Earth's surface was the same force responsible for the moon's motion above and around Earth. Newton hypothesized that Earth's gravity extends that far into space and perhaps even farther.

This force due to gravity is also called weight. The weight of an object depends on:

the mass of the object

$\vec{F}_{\text{g}} \propto m$

and the acceleration due to gravity

$\vec{F}_{\text{g}} \propto g$

$\vec{F}_{\text{g}} = mg$

The acceleration due to gravity, *g*, near Earth's surface is approximately $9.81\,\text{m/s}^2$, but this is not a constant value. This value depends on the mass of the Earth and the distance of the object experiencing the force due to gravity from Earth's centre.

$g \propto m$

$g \propto \frac{1}{d^2}$

Earth is not a perfect sphere (ball). It is somewhat flatter at the poles, and somewhat bulged at the equator. For this reason, the acceleration due to gravity on Earth's surface varies with the latitude. The value of *g* is slightly less at the equator than it is at the poles. Similarly, the acceleration due to gravity near the surface of another planet or object in space can be measured if you know the mass of the object and you can measure the weight of that object near that planet.

NOTES

Example

What is the weight of a 12.0 kg object near the surface of Earth?

Solution

$$\vec{F}_g = m\vec{g}$$
$$= (12.0\ \text{kg})(-9.81\ \text{m/s}^2)$$
$$= -118\ \text{N or } 118\ \text{N down}$$

The value of force is negative because the force points down toward the surface of Earth. The weight of the object is 118 N.

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PRACTICE EXERCISES

Formula: $\vec{F}_g = m\vec{g}$

1. What is the weight of a 25.0 kg object near Earth's surface?

2. What is the mass of an object if it has a weight of 80.0 N near Earth's surface?

3. What is the acceleration due to gravity near the surface of the moon if an object that has a mass of 22.0 kg has a weight of 36.0 N near the moon's surface?

4. What is the weight of a 72.0 kg object near Earth's surface?

5. What is the mass of an object if it has a weight of 127 N near Earth's surface?

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Lesson 4 CONTACT FORCES

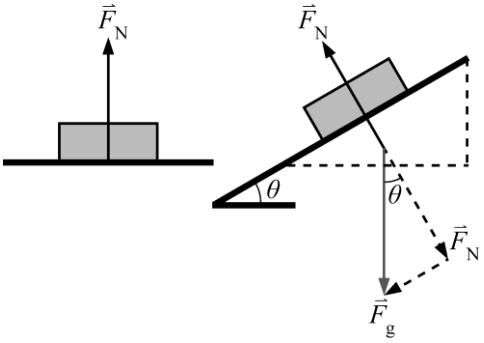
NOTES

People experience forces acting upon them and objects all around them every day. As previously discussed, the net force on an object is the sum of all forces acting on the object. These various forces are:

- normal force (F_N)
- frictional force (F_f)
- applied force (F_a)
- tension (T)
- elastic force (F_s)

NORMAL FORCE

The *normal force* is the supporting force exerted by a lower surface on an upper surface. When an object is on a surface, the surface exerts an upward force on the object. This force is the normal force and acts perpendicular to the surface.



For simplicity, only normal and frictional forces from a horizontal surface will be examined. On a horizontal surface, the normal force is equal in magnitude and opposite in direction to the weight of the object. Consider a motionless box on a horizontal surface. Since the box is motionless, the net force on the box is zero. The sum of all forces on the box is:

$$\vec{F}_{net} = \vec{F}_N + \vec{F}_g = 0$$

Choose upward as the positive direction:

$\vec{F}_N + \vec{F}_g = 0$ (sub in your magnitudes with upward being positive)

$$F_N - mg = 0$$
$$F_N = mg$$

This means that on a horizontal surface the normal force has a magnitude equal to *mg* and points upward. Remember, the force of gravity also has a magnitude of *mg* but it points downward.

$\vec{F}_N = mg$ up
and
$\vec{F}_g = mg$ down

NOTES

Example

A 7.6 kg object is resting on a horizontal surface. What is the normal force acting on the object?

Solution

The normal force points upward and is equal in magnitude to the parallel component of the weight.

$F_f = \mu F_N$

$$\begin{aligned}\vec{F}_N &= -m\vec{g} \\ &= -(7.6\text{ kg})(-9.81\text{ m/s}^2) \\ &= 75\text{ N up}\end{aligned}$$

FRICTIONAL FORCE

Frictional forces are forces that always oppose motion. They depend on:

- the magnitude of the normal force
- the nature of the two surfaces

NOTE: Frictional forces do not depend on the area of contact between the two surfaces.

The nature of the surfaces is expressed as the coefficient of friction. The coefficient of friction is symbolized by the Greek letter mu (μ) and has no units.

You may have noticed that it takes more force to start an object moving along a surface than it takes to keep it moving once it has started. The reason for this is that there is a coefficient of static friction, μ_s, and a coefficient of kinetic friction, μ_k. The coefficient of static friction is greater than the coefficient of kinetic friction. The value of the coefficient of static and kinetic friction will change depending on the two surfaces that are in contact. Rougher surfaces like sand paper have a high coefficient of friction while smooth surfaces like ice have a very low coefficient of friction.

$\mu_s > \mu_k$

$F_f = \mu F_N$

Notice that in this equation there are no vector quantities. This equation finds the magnitude of the force of friction and is directly proportional to the magnitude of the normal force. The direction must be decided upon by considering the motion of the object. The direction of the force of friction will always be parallel to the surface and opposite to the direction of motion.

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NOTES

Example

A 7.6 kg object is pulled along a horizontal surface. If the coefficient of friction between the surfaces is 0.20, what is the force of friction?

Solution

$$\begin{aligned}\vec{F}_N &= -m\vec{g} \\ &= -(7.6\text{ kg})(-9.81\text{ m/s}^2) \\ &= 74.6\text{ N up} \\ F_f &= \mu_k F_N \\ &= (0.20)(74.6\text{ N}) \\ &= 15\text{ N}\end{aligned}$$

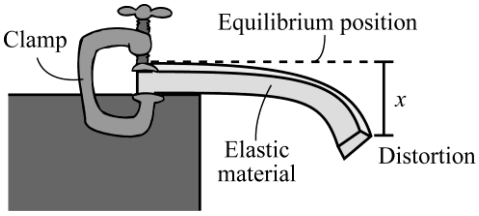
TENSION AND APPLIED FORCE

When you push on a car to get it rolling, push a box across the floor or shoot a hockey puck into a net you are applying force to it. The symbol for an applied force is F_a, and it is measured in newtons (N) because it is a force.

Tension is similar to applied force. It is the pulling force exerted indirectly to an object through a rope, string, chain, etc. Examples of two of the most common types of tension forces are a person pulling an object with a rope and a cord holding up a hanging lamp. The symbol for tension is *T* and, because it is a force, it is measured in newtons (N). The direction of a tension force is always along the line of the rope, chain, string, etc. and points away from the object.

ELASTIC FORCE

When you stretch an elastic band, it will restore itself when you remove the force applied to it. When you stretch or compress a spring by applying a force on it, the spring will restore itself to its original position when the applied force is removed. The force exerted within these materials to restore it to its original position is called the elastic force (F_s). The original position of the elastic material is called the *equilibrium position*.
The displacement (x) is the change in position relative to the equilibrium position.



NOTES

Hooke's law:

$\vec{F}_s = -k\vec{x}$

Hooke's law states that the elastic force is directly proportional to the displacement (x) of the material.

$F_s \propto x$ or $\vec{F}_s = -k\vec{x}$

where k is a proportionality constant called the spring constant.

The elastic force is negative because it acts in the opposite direction of the displacement (x). Every elastic material (a spring, elastic band, etc.) has a spring constant. The stiffer the elastic material is, the greater the spring constant.

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PRACTICE EXERCISES

Formula: $F_f = \mu F_N$ $\vec{F}_N = -m\vec{g}$ $\vec{F}_{net} = m\vec{a}$ $\vec{F}_s = -k\vec{x}$

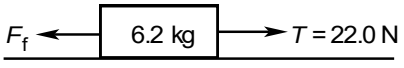
1. A 14.0 kg object is resting on a horizontal surface. What is the normal force acting on the object?

2. A 9.6 kg object is pulled along a horizontal surface. If the coefficient of friction between the surfaces is 0.11, what is the magnitude of the force of friction?

3. A 20.0 N object is placed on a horizontal surface. A force of 3.0 N is required to keep the object moving at a constant speed, what is the coefficient of friction between the two surfaces?

4. A 16.2 kg object is pulled along a frictionless horizontal surface by a horizontal net force of 10.2 N. What is the normal force acting on the object?

5. A 6.2 kg object is pulled to the right along a horizontal surface, as shown in the diagram, by an applied force of 22.0 N. If the acceleration of the object is 1.1 m/s^2, what is the coefficient of kinetic friction between the surfaces?



6. A 1.00 kg mass on a spring is 0.100 m to the right from its equilibrium position. If the spring constant is 20.0 N/m, what is the elastic force acting on the mass?

7. The restoring force acting on a 0.50 kg object on a spring is 2.0 N leftward. If the spring constant is 15 N/m, what is the distortion of the object?

8. The elastic force acting on a 0.60 kg object on a spring is 1.2 N. If the distortion of the object is 0.025 m to the right, what is the spring constant?

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9. A weight of 1.65 N will stretch a vertical spring 0.110 m. What is the spring constant?

10. A mass of 5.0 kg will stretch a vertical spring 3.25 cm. What is the spring constant?

11. A weight of 9.3 N is hung on a vertical spring that has a spring constant of 25 N/m. How far will the spring stretch?

12. A 75 g object vibrates at the end of a spring (k = 5.0 N/m). If the maximum displacement of the object is 0.080 m to the right, what is the maximum acceleration?

Lesson 5 NET FORCE AND MOTION

NOTES

The net force on an object is the vector sum of all the forces acting on the object.

The net force on an object is the vector sum of all the forces acting on the object. When the sum is zero, the forces are balanced. When the forces are unbalanced, the object will be accelerated in proportion to the net force.

To determine the net force on an object, all the forces acting on the object must be clearly identified. A free-body diagram is useful for identifying all of the forces acting on a body.

FREE-BODY DIAGRAMS

In a free-body diagram, the object is represented by a simple geometric shape, or even a point. All of the acting forces are sketched with appropriate directions and a length that is roughly proportional to the magnitude of the force.

By convention, the forces are drawn as though they are acting through the object's centre of mass. This is usually the case. Friction, however, acts along the surface, and because the forces in a real system are not concurrent (acting through a single point), there may be a tendency for some rotational motion. For simplicity, assume that all forces illustrated in free-body diagrams are concurrent.

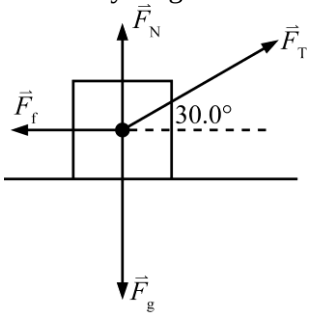
Example

An object is pulled at constant velocity along a horizontal surface. The force is applied through a rope that makes an angle of 30.0° to the horizontal.

It can be useful to draw a free-body diagram to determine the net force.

Free-body diagrams show all the forces acting on the object.

Free-body diagram:



Because actual values for the forces are unspecified, you can only be certain about the fact that the magnitude of the normal force and the weight are equal. These vectors have been drawn with the same length.

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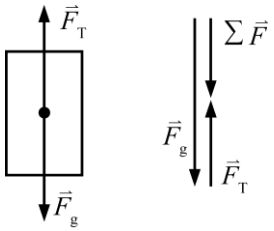
NOTES

Example

An elevator with a mass of 9.00×10^2 kg is accelerating downward at a rate of $1.30\ \text{m/s}^2$. What is the tension in the cable?

Solution

Draw a free-body diagram. Because the elevator is accelerating downward, the net force must also act downward.



Consider up as the positive direction.

Find $\vec{F}_g$.

$$\begin{aligned}\vec{F}_g &= m\vec{g}\\ &= \left(9.00\times10^2\ \text{kg}\right)\left(-9.81\ \text{m/s}^2\right)\\ &= -8.83\times10^3\ \text{N (down)}\end{aligned}$$

Find $\sum\vec{F}$.

$$\begin{aligned}\sum\vec{F} = \vec{F}_{net} &= m\vec{a}\\ &= \left(9.00\times10^2\ \text{kg}\right)\left(-1.30\ \text{m/s}^2\right)\\ &= -1.17\times10^3\ \text{N (down)}\end{aligned}$$

Find $\vec{F}_T$.

The net force is the vector sum of all the forces acting on the elevator.

$$\begin{aligned}\vec{F}_{net} &= \Sigma\vec{F} = \vec{F}_g + \vec{F}_T\\ \vec{F}_T &= \Sigma\vec{F} - \vec{F}_g\\ &= \left(-1.17\times10^3\ \text{N}\right) - \left(-8.83\times10^3\ \text{N}\right)\\ &= 7.66\times10^3\ \text{N (up)}\end{aligned}$$

Remember to include the negative in both the $\sum\vec{F}$ and $\vec{F}_g$.

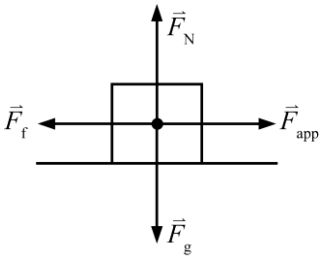
NOTES

Example

An object that has a mass of 25.0 kg is pushed along a horizontal surface with a force of 95.0 N. If the force of friction is 50.0 N, what is the acceleration of the object?

Solution

Draw a free-body diagram. In this case, it is useful to separate the net force acting in the vertical direction (which is zero) from the net force acting in the horizontal direction (which is clearly greater than zero).



The object is accelerating horizontally. Therefore, you only need to consider the horizontal forces.

Find $\sum \vec{F}$.

$$\sum \vec{F} = \vec{F}_{net} = \vec{F}_{app} + \vec{F}_f$$
$$= 95.0 \text{ N} + (-50.0 \text{ N})$$
$$= 45.0 \text{ N}$$

Find the acceleration.

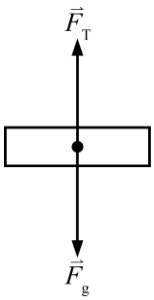
$$\sum \vec{F} = m\vec{a}$$
$$\vec{a} = \frac{45.0 \text{ N}}{25.0 \text{ kg}}$$
$$= 1.8 \text{ m/s}^2 \text{ (to the right)}$$

Example

A 75.0 kg student stands on a bathroom scale in an elevator that is accelerating upward at a rate of 1.00 m/s^2. What is the reading (in newtons) on the scale? (Note: The scale reading will be the magnitude of the applied force, $\vec{F}_T$.)

Solution

Draw a free-body diagram.



Consider up as the positive direction.

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NOTES

Find $\sum \vec{F}$.

$$\begin{aligned}\sum \vec{F} &= m\vec{a} \\ &= (75.0 \text{ kg})(1.00 \text{ m/s}^2) \\ &= 75.00 \text{ N (up)}\end{aligned}$$

Find $\vec{F}_g$.

$$\begin{aligned}\vec{F}_g &= m\vec{g} \\ &= (75.0 \text{ kg})(-9.81 \text{ m/s}^2) \\ &= -735.8 \text{ N}\end{aligned}$$

Find the magnitude of $\vec{F}_T$ (scale reading).

$$\begin{aligned}\sum \vec{F} &= \vec{F}_T + \vec{F}_g \\ \vec{F}_T &= \sum \vec{F} - \vec{F}_g \\ &= 75.00 \text{ N} - (-735.8 \text{ N}) \\ &= 811 \text{ N (up)}\end{aligned}$$

The scale reads 811 N.

PRACTICE EXERCISES

Formulas:

$\sum \vec{F} = m\vec{a}$ $\qquad$ $F_N = F_g \cos\theta$ or $F_N = mg\cos\theta$

$\vec{F}_g = m\vec{g}$ $\qquad$ $F_f = \mu F_N$ or $F_f = \mu mg\cos\theta$

1. An 11.0 kg object is thrown vertically into the air with an applied tensional force of 145 N. What is the magnitude of the initial acceleration of the object?

2. A 12.0 kg object is pushed with a horizontal force of 6.0 N due east across a horizontal table. If the force of friction between the two surfaces is 2.0 N, what is the acceleration of the object?

3. A 20.0 kg object is pulled horizontally with a rope along a level floor with an applied force of 27.0 N. If this object is accelerating at a rate of 0.80 m/s^2 what is the magnitude of the force of kinetic friction?

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4. A 6.3 kg object is thrown upward with an acceleration of 0.45 m/s^2. What is the magnitude of the force applied to the object?

5. An elevator has a mass of 1.20×10^3 kg .

a) What is the tension in the cable if the elevator is accelerating downward at a rate of 1.05 m/s^2.

b) The elevator is accelerating upward at a rate of 1.05 m/s^2.

c) The elevator is moving downward at a constant velocity of 1.10 m/s.

6. An object that has a mass of 36.0 kg is pushed along a horizontal surface with a force of 85.0 N. If the force of friction is 72.0 N, what is the magnitude of the acceleration of the object?

7. A 1.0 kg object is given a push along a horizontal surface. The velocity of the object when it is released is 0.50 m/s west, and the object slides 0.25 m before coming to a stop. What is the magnitude of the force of kinetic friction?

8. A 100 N box is propelled north along a horizontal surface by a 250 N horizontal force. If the force of friction on the box is 140 N south, what is the acceleration of the box?

9. A 7.00 kg object rests on a horizontal frictionless surface. What is the magnitude of the horizontal force that is required to accelerate it at the rate of 2.30 m/s^2 north?

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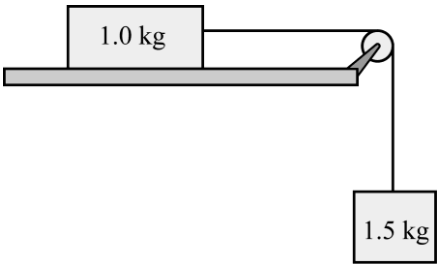
10. A car is travelling at a velocity of 24.0 m/s east when the brakes are applied, causing the wheels to lock. The force of kinetic friction between the tires and the road surface is 1.80×10^4 N. If the mass of the car and occupants is 1.50×10^3 kg, how far does the car skid before coming to a stop?

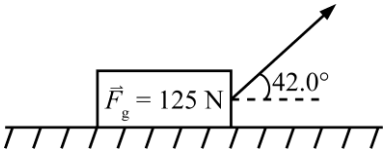
11. A 1.20×10^3 kg car is travelling at a velocity of 20.0 m/s east when its brakes are locked. Assuming a force of kinetic friction of 2.50×10^4 N, what is the velocity of the car after 0.500 s?

12. A box with a mass of 1.0 kg is resting on a horizontal, frictionless surface. It is accelerated by attaching a 1.5 kg mass, as shown in the diagram.



Assume that the pulley is also frictionless and that the cord has negligible mass. Remember: The system undergoing acceleration consists of both attached masses.What is the acceleration of the box?

13. A 125 N box is pulled east along a horizontal surface with a force of 60.0 N acting at an angle of 42.0°, as shown in the diagram.



If the force of friction on the box is 15.0 N, what is the acceleration?

14. A 725 N student stands on a bathroom scale while riding in an elevator. The student observes that the scale reads 775 N as the elevator begins to rise. Remember, the scale reading is the applied force, $\vec{F}_T$ .Find the acceleration of the elevator as it begins to rise.

15. A hockey puck with a mass of 0.48 kg is shot north along the ice with an initial velocity of 3.0 m/s. If the puck comes to rest after travelling 8.0 m, what is the force of friction on the puck?

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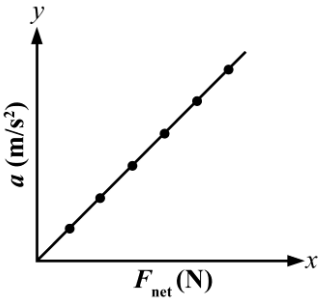
PRACTICE TEST

1. Explain what the coefficient of friction, μ, physically represents.

2. How is inertia related to the mass of an object?

3. Charlie obtained the following graph when studying the relationship between the acceleration of an object and the net force acting on the object.



What did the graph tell Charlie about the relationship between the two variables?

4. Sophia stands motionless on the surface of Earth.

a) What forces act on her?

b) If forces are acting on the student, why does she remain motionless?

5. What is the mass of an object that has a weight of 50.0 N near Earth's surface?

6. A hockey puck is shot along the ice at a velocity of 11 m/s, and it slides 25 m before it comes to a stop. What is the coefficient of kinetic friction between the puck and the ice?

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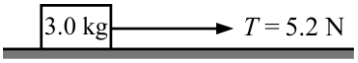
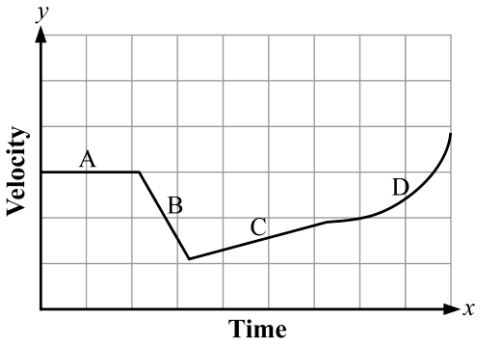
7. Two 100 kg objects sit 10.0 metres apart. What direction is the force of gravity between these two objects?

8. A 3.0 kg object is pulled along a horizontal surface as shown in the diagram by a horizontal force of 5.2 N. If the object is accelerating at a rate of 1.2 m/s^2 to the right, what is the force of friction acting on the object?



9. A 4.0 kg object is sliding along a surface to the right and eventually comes to a stop due to a frictional force of 8.0 N. How do the magnitude and direction of the force of friction change during motion and after the object has stopped?

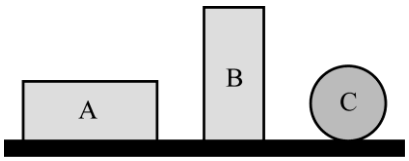
10. The velocity-time graph of an object moving along a horizontal surface.



a) In which section of the graph does the object experience the largest constant net force? Explain.

b) In which section of the graph is the applied force equal in magnitude to the force of kinetic friction? Explain.

11.



A, B, and C are three objects with identical masses at rest on a horizontal surface. On which of the above objects is/are the normal force the greatest?

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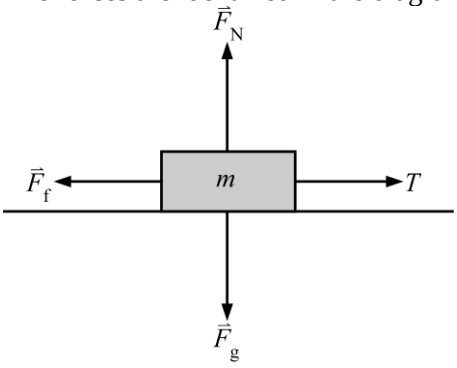
12. An object is pulled along a horizontal surface at a constant velocity as shown in the diagram. The forces are identified in the diagram.



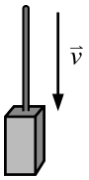
Why is this object not accelerating?

13. Two boys engage in a tug of war, exerting a force of 250 N at opposite ends of a sturdy rope.

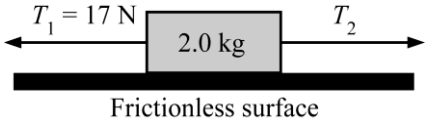
a) Explain how Newton's third law applies to this system. Why is the vector sum of the action force and the reaction force not a sensible quantity?

b) Explain how Newton's second law applies to this system. What is the vector sum of the forces?

14. A 5.0 N object is tied to a string and moves downward at a constant velocity. What is the tension in the string?

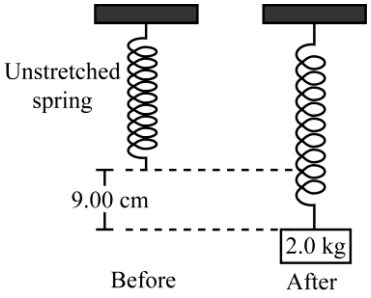


15. Two forces, T_1 and T_2, are exerted on a 2.0 kg object as shown in the diagram.



If T_1 = 17 N and the acceleration of the object is 1.6 m/s^2 to the left, what is the magnitude of T_2?

16. When a 2.00 kg mass is hung on a vertical spring, the spring is stretched 9.00 cm as shown in the diagram and sits motionless. What is the spring constant of this spring?



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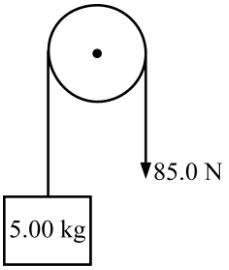
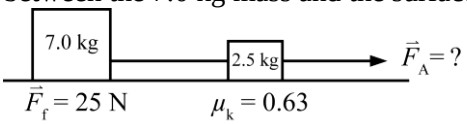
17. A constant force of 85.0 N acts straight down, as shown in the given diagram.



Assuming the pulley has negligible mass and is frictionless, how long will it take to lift the 5.00 kg mass a distance of 1.00 m? The mass was initially at rest.

18. Two masses resting on a horizontal surface are connected with a cord. The force of kinetic friction between the 7.0 kg mass and the surface is a constant 25 N.

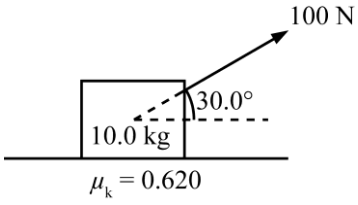


a) Calculate the coefficient of kinetic friction between the 7.0 kg mass and the surface.

b) If the system accelerates at 2.5 m/s^2, what force must be applied to the system?

19. A student throws a 1.50 kg object straight up into the air accelerating it at a rate of 1.20 m/s^2. What is the force that the student applies to this object?

20. A 10.0 kg mass is resting on a horizontal surface. A force of 100 N acts at an angle of 30.0° to the horizontal.



a) Calculate the normal force.

b) Calculate the magnitude of the force of friction.

c) What is the acceleration of the mass?

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ENERGY

BIG IDEA

Energy is found in different forms, is conserved, and has the ability to do work.

As you begin this unit, you should consider the following questions…

- What is the relationship between work, energy, and power in a system?
- How are the conservation laws applied in parallel and series circuits?
- Why can't a machine be 100% efficient?
- How is specific heat capacity determined?
- What explanations can you offer when your experimental data show that energy is not conserved?
- What are some possible innovations that might replace the internal combustion engine?

Lesson	Page	Completed on
1. Work	144	
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3. Kinetic Energy	156	
4. Law of Conservation of Energy	160	
5. Power	169	
6. Machines and Efficiency	175	
7. Electric Current	180	
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Answers and Solutions	at the back of the book	

When you are finished this unit, you are expected to know …

- The **conservation of energy**; **principle of work**, and **energy**
- How to perform calculations involving **power and efficiency**, including **mechanical and electrical** power
- How to determine the efficiency of **simple machines** (such as the **lever, ramp, wedge, pulley, screw, and the wheel and axle**) and understand its role in determining the **mechanical advantage** of such machines.
- How **electric circuits (DC)** work, and the application of both **Ohm's law and Kirchhoff's laws**, **including terminal voltage versus electromotive force (emf)**

  How **thermal equilibrium and specific heat capacity is an application of the law of conservation of energy**

Lesson 1 WORK

NOTES

Energy is defined as the ability to do work.

The concept of energy was developed during the mid 1800s and is still a very common concept in modern society. Energy can be used to describe a person, someone can be energetic or tired, they have a lack of energy for work. Energy also exists in many forms such as electricity, gasoline, wind, the sun, etc. In physics, *energy* is defined as the ability to do work.

The concept of work means different things to different people. In this book, work is defined as the magnitude of the force multiplied by the magnitude of the displacement.

$$\text{work} = \text{force} \times \text{displacement}$$
$$W = Fd$$

Since work is the product of the magnitudes of the force and displacement, it is a scalar quantity and is measured in newton metres $(\text{N} \cdot \text{m})$ or joules (J). Work can be a negative or positive quantity. When work is done in the direction of motion it is always positive. When work is done is the opposite direction of motion, it will be negative. The amount of work done can also be found graphically. On a force-displacement graph, work is the area under the curve.

It is important to note the language that is used in physics to describe work. In physics, one talks about the amount of work that is done *on an object*. If you are standing holding a 30.0 kg object, you are using energy. Therefore, you are doing work. However, this work is not done on the object; it is done on your muscles by a force acting to contract these muscles. In this course, only the work done *on an object* will be discussed. If the object does not move through some displacement, there is no work done on the object. In the same way, if there is no force acting on the object, there is no work. Work is either done to overcome an opposing force (force of gravity, friction, etc.) on the object, or to accelerate an object.

$W = Fd$

When an object is lifted at a constant velocity, work is done against gravity. The applied force to lift the object at a constant velocity will be equal in magnitude to the force of gravity. This means the direction of the force is upward and therefore it has a positive value. In this situation, work is related to the force of gravity and is expressed as:

$W = F_{\text{a}} d$ and $\vec{F} = m\vec{g}$

As mentioned above, the applied force must push in the opposite direction as the force of gravity to hold the object at constant velocity. This means that:

$\vec{F}_{\text{a}} = m\vec{g}$ where $\vec{g} = 9.81 \text{ m/s}^2$ up

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NOTES

Since work is the product of the magnitudes of the force and displacement:

$W = F_a d$

$W = mgd$ or $W = mgh$

where $m =$ mass

$g =$ magnitude of the acceleration due to gravity $\left(9.81 \text{ m/s}^2\right)$

$h =$ height (vertical displacement)

The variable for vertical displacement is often written as either *h* or *d*. In the practice exercises to follow, the variable *h* will always be used for vertical displacement.

$W = mgd$ or $W = mgh$

Example

A 15.0 kg object is lifted at constant velocity from the floor to a height of 1.50 m. How much work is done on the object?

Solution

$W = mgh$

$= (15.0 \text{ kg})\left(9.81 \text{ m/s}^2\right)(1.50 \text{ m})$

$= 221 \text{ J}$

Example

A 10.0 kg object is moved horizontally 5.00 m across a level floor using a horizontal force of 3.00 N. How much work is done on the object?

Solution

$W = Fd$

$= (3.00 \text{ N})(5.00 \text{ m})$

$= 15.0 \text{ J}$

Example

A 3.0 kg object is held 1.2 m above the floor for 15 s. How much work is done on the object?

Solution

$W = mgh$

$= (3.0 \text{ kg})\left(9.81 \text{ m/s}^2\right)(0)$

$= 0$

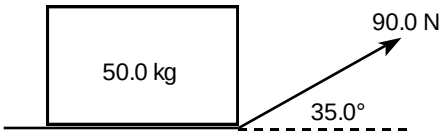
In some cases, the force might not be parallel to the displacement. When the force is not in the same direction as the displacement, only the component of the force that is in the direction of displacement is used.

NOTES

Example

A 50.0 kg box is pulled 11.0 m along a level surface by a rope. If the rope makes an angle with the surface of 35.0° and the force exerted through the rope is 90.0 N, how much work is done on the box?

Solution



Since the object moves horizontally, find the horizontal component of the force parallel to the direction of displacement.

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\begin{aligned}\text{adjacent} &= (\cos\theta)(\text{hypotenuse}) \\ &= (\cos 35.0°)(90.0\text{ N}) \\ &= 73.72\text{ N}\end{aligned}$$

$$\begin{aligned}W &= Fd \\ &= (73.72\text{ N})(11.0\text{ m}) \\ &= 811\text{ J}\end{aligned}$$

Example

A 1385 kg car travelling at 61 km/h is brought to a stop while skidding 42 m. What is the work done on the car by the frictional forces?

Solution

Convert 61 km/h to m/s:

$$\vec{v}_0 = 61\text{ km/h} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ h}}{3600\text{ s}} = +16.9\text{ m/s}$$

$$\begin{aligned}\vec{v}_f^{\,2} &= \vec{v}_0^{\,2} + 2\vec{a}\vec{d} \\ -(16.9\text{ m/s})^2 &= 2\vec{a}(42\text{ m}) \\ \vec{a} &= -3.40\text{ m/s}^2\end{aligned}$$

$$\begin{aligned}\vec{F} &= m\vec{a} \\ &= (1385\text{ kg})(-3.40\text{ m/s}^2) \\ &= -4.71\times10^3\text{ N}\end{aligned}$$

$$\begin{aligned}W &= Fd \\ &= (-4.71\times10^3\text{ N})(42\text{ m}) \\ &= -2.0\times10^5\text{ J}\end{aligned}$$

NOTE: The answer is negative because the force and the displacement are in opposite directions.

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PRACTICE EXERCISES

Formulas: $W = Fd$ $W = mgh$

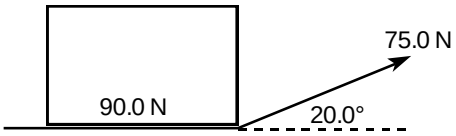
1. A 20.0 N object is lifted at a constant velocity from the floor to a height of 1.50 m. How much work is done on the object?

2. A 15.0 N object is moved horizontally 3.00 m across a level floor using a horizontal force of 6.00 N. How much work is done on this object?

3. A 2.20 N object is held 2.20 m above the floor for 10.0 s. How much work is done on the object?

4. A 10.0 kg object is accelerated horizontally from rest to a velocity of 11.0 m/s in 5.00 s by a horizontal force. How much work is done on this object assuming the object is on a frictionless surface?

5.



A 90.0 N box is pulled 10.0 m along a level surface by a rope. If the rope makes an angle of 20.0° with the surface, and the force exerted through the rope is 75.0 N, how much work is done on the box?

6. A 60.0 kg student runs at a constant velocity up a flight of stairs. If the vertical distance of the stairs is 3.2 m, what is the work done against gravity?

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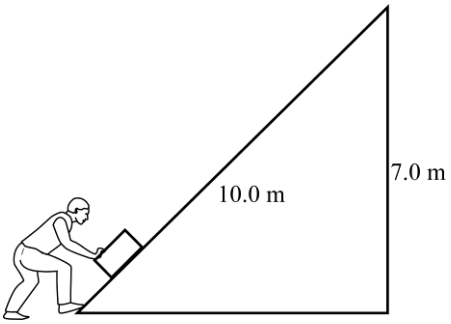
7. A 20.0 kg box is pulled horizontally 9.0 m along a level frictionless surface at a constant velocity. How much work is done on the box?

8.

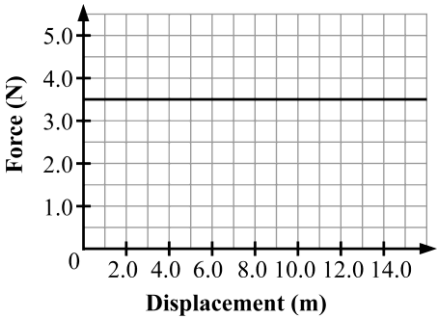


An 80.0 kg box is pushed at a constant velocity along a frictionless incline as shown in the diagram. How much work is done on the box in moving it from the bottom to the top of the incline?

9. A 25.0 kg object is accelerated from rest through a distance of 6.0 m in 4.0 s across a level floor. If the force due to friction between the object and the floor is 3.8 N, what is the work done in moving the object?

10. A 1165 kg car travelling at 55 km/h is brought to a stop while skidding 38 m. Calculate the work done on the car by the frictional forces.

11. Given the following force-displacement graph of an object being pulled along a level surface, determine the work done in moving the object 16.0 m.



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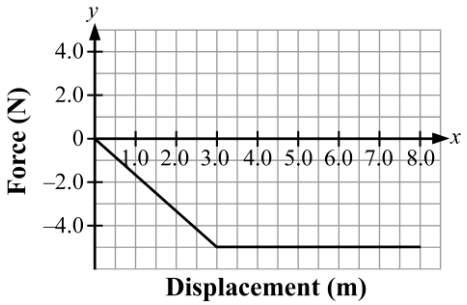
12. Given the following force-displacement graph of an object moving along a horizontal surface, determine the work done in moving the object 8.0 m.



Lesson 2 ENERGY

NOTES

There are many forms of energy: mechanical energy, thermal energy, electrical energy, nuclear energy, etc. Each form of energy can be converted to another form. For example, mechanical energy can be converted to electrical energy using an electric generator. When energy is changed to different forms, work is done.

Mechanical energy
• potential energy
• kinetic energy

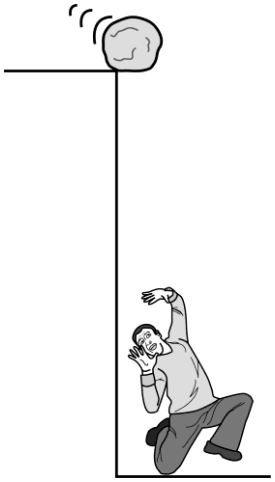
In this course, mechanical energy will be examined. Mechanical energy is the energy of an object because of its position, or because of its motion. There are two types of mechanical energy: potential energy and kinetic energy. *Potential energy* is the stored energy of an object due to its position. *Kinetic energy* is the energy of an object due to its motion. When one energy component is converted to another, work is done. Work is the change in energy of the system. Written mathematically:

$$W = \Delta E$$

GRAVITATIONAL POTENTIAL ENERGY

Potential energy is stored energy due to an objects' position.

This boulder has potential energy as a result of its position. It has the potential to fall to the ground, and do work on the ground (that is, deform or damage the ground). This is gravitational potential energy or potential energy due to gravity.



When work is done against gravity, this work is stored in the system as gravitational potential energy.

The gravitational potential energy of an object depends on:
- the force acting on the object
- the displacement (change in position) of the object

The potential energy stored in an object by doing work on the object is:

$$\text{change in potential energy} = \text{force} \times \text{displacement}$$

$$\Delta E_{\text{p}} = F \Delta d$$

For gravitational potential energy:
$\Delta E_{\text{p}} = mg\Delta h$

As shown in the previous section, when dealing specifically with gravitation potential energy, the definition of the force of gravity can be used with this equation to yield:

$$\Delta E_{\text{p}} = mg\Delta h$$

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NOTES

Potential energy is a scalar quantity. Since it is the product of force and displacement, it is measured in newton metres (N·m) or joules (J). Potential energy can also be a negative or positive quantity. When there is a loss in potential energy, the value will be negative. When there is a gain in potential energy, the value will be positive.

You will note that these are the same equations that were used for work. This should be of no surprise because the gravitational potential energy of an object is changed by doing work on the object against gravity.

Gravitational potential energy is always relative to some reference point. For example, if you hold an object above a table, the gravitational potential energy may be determined with respect to the table top, the floor, or any other reference point.

Example

A 15.0 kg object is lifted from the floor to a vertical height of 2.50 m. What is the potential energy (gravitational) of the object with respect to the floor?

Solution

$$\begin{aligned} E_p &= mgh \\ &= (15.0\ \text{kg})(9.81\ \text{m/s}^2)(2.50\ \text{m}) \\ &= 368\ \text{J} \end{aligned}$$

NOTE: Gravitational potential energy is usually expressed in relation to some point. (e.g. with respect to the ground or sea level)

Example

An archer pulls on the bow string with an average force of 12.0 N while drawing the arrow back a distance of 2.00×10^{-1} m. Calculate the potential energy of the bow-arrow system.

Solution

$$\begin{aligned} E_p &= Fd \\ &= (12.0\ \text{N})(2.00 \times 10^{-1}\ \text{m}) \\ &= 2.40\ \text{J} \end{aligned}$$

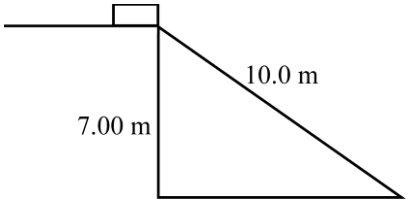
PRACTICE EXERCISES

Formulas: $E_p = Fd$ $E_p = mgh$

1. A 25.0 N object is held 2.10 m above the ground. What is the potential energy of the object with respect to the ground?

2. An uncompressed spring is 20.0 cm in length. What is the potential energy of this spring when an average force of 65.0 N compresses it to a length of 13.5 cm?

3.



A 2.75 kg box is at the top of a frictionless incline as shown in the diagram. What is the potential energy of the box with respect to the bottom of the incline?

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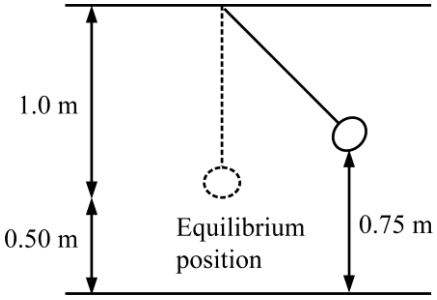
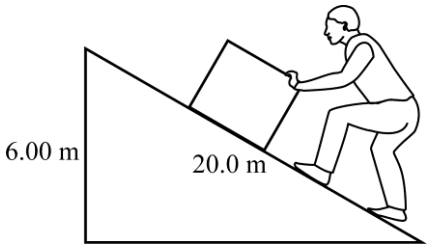
4.



The bob of a pendulum has a mass of 2.0 kg. This bob is pulled sideways so that it is 0.75 m above the table top. What is the potential energy of the bob with respect to the equilibrium position?

5.



A 2.00×10^2 kg object is pushed to the top of an incline as shown in the diagram. If the force applied along the incline is 6.00×10^2 N, what is the potential energy of the object when it is at the top of the incline with respect to the bottom of the incline?

Lesson 3 KINETIC ENERGY

NOTES

When an object is thrown, a force is applied and this force acts through some distance. Therefore, work is done.

$W = Fd$

Kinetic energy is energy in motion.

However, when the object is released, it has a speed; it has kinetic energy. Kinetic energy is energy of motion. The work done in accelerating this object from rest becomes kinetic energy.

$$E_k = \frac{1}{2}mv^2$$

This equation is related to the equation for work. Consider an object at rest that is accelerated horizontally by an applied force through some distance along a frictionless surface. Using kinematics equations and Newton's second law:

$$\vec{v}_f^{\,2} = \vec{v}_0^{\,2} + 2\vec{a}\vec{d} \quad (\vec{v}_0 = 0)$$

$$\vec{v}_f^{\,2} - \vec{v}_0^{\,2} = 2\vec{a}\vec{d} \text{ subbing in } \vec{a} = \frac{\vec{F}}{m}$$

$$\vec{v}_f^{\,2} - \vec{v}_0^{\,2} = \frac{2Fd}{m}$$

When you multiply two vectors together that are in the same direction, the result is a scalar value. Because of this, the equation can be written in terms of magnitudes:

$$v_f^2 - v_0^2 = \frac{2Fd}{m}$$

In this case, there is a change in kinetic energy because the work being done is putting the object into motion from rest. Therefore, the change in energy will be equal to the work done:

$$\Delta E = \Delta E_k = W = Fd$$

$$\therefore v_f^2 - v_0^2 = \frac{2Fd}{m} \text{ becomes}$$

$$v_f^2 - v_0^2 = \frac{2\Delta E_k}{m}$$

$$\Delta E_k = \frac{1}{2}m\left(v_f^2 - v_0^2\right)$$

Since the object started from rest ($v_0 = 0$), this can also be written as:

$$E_k = \frac{1}{2}mv^2$$

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NOTES

Like potential energy, kinetic energy is a scalar quantity measured in joules (J). The change in kinetic energy will have a negative value if there is a loss in kinetic energy (an object slows down) and have a positive value if there is a gain in kinetic energy (an object speeds up). Note that the kinetic energy of an object is always positive (or zero).

Example

A 60.0 kg student is running at a uniform speed of 2.70 m/s. What is the kinetic energy of the student?

Solution

$$\begin{aligned} E_k &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(60.0\text{ kg})(2.70\text{ m/s})^2 \\ &= 219\text{ J} \end{aligned}$$

Example

The kinetic energy of a 2.1 kg object is 1.00×10^3 J. What is the speed of this object?

Solution

$$\begin{aligned} E_k &= \frac{1}{2}mv^2 \\ v &= \sqrt{\frac{2E_k}{m}} \\ &= \sqrt{\frac{2(1.00\times10^3\text{ J})}{2.10\text{ kg}}} \\ &= 31\text{ m/s} \end{aligned}$$

PRACTICE EXERCISES

Formula: $E_k = \frac{1}{2}mv^2$

1. A 3.0 kg object is travelling at a constant speed of 7.5 m/s. What is the kinetic energy of this object?

2. The kinetic energy of a 20.0 N object is 5.00×10^2 J. What is the speed of this object?

3. A 10.0 N object is accelerated uniformly from rest at a rate of 2.5 m/s^2. What is the kinetic energy of this object after it has accelerated a distance of 15.0 m?

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4. An 8.0 kg object is dropped from a height of 7.0 m. What is the kinetic energy of this object as it hits the ground?

5. A 10.0 N object has kinetic energy of 3.00×10^2 J. What is the speed of the object?

Lesson 4 LAW OF CONSERVATION OF ENERGY

NOTES

The law of conservation of energy states that energy cannot be created nor destroyed, but it can be changed from one form to another. That is, the total energy of an isolated system remains constant.

Energy is changed from one form to another when work is done on an object.

$W = \Delta E = Fd$

where F = magnitude of the force

d = magnitude of displacement

- When work is done on an object to accelerate it, the kinetic energy of the object changes.

  $E_k = \frac{1}{2}mv^2$

- When work is done on an object to change the vertical position (work done against gravity), the gravitational potential energy of the object changes.

  $E_p = mgh$

- When work is done on an object to overcome friction, the thermal energy of the object increases. The kinetic or potential energy of the system is lost to heat energy.

  $E_h = F_f d$

LAW OF CONSERVATION OF MECHANICAL ENERGY

Both kinetic and potential energy are considered types of mechanical energy. If the only forces acting on an object are conservative forces, the mechanical energy of the object does not change—this is known as the law of conservation of mechanical energy. A force is conservative if the work done in moving an object from one point to another against the force does not depend on the path taken. The mechanical energy is conserved.

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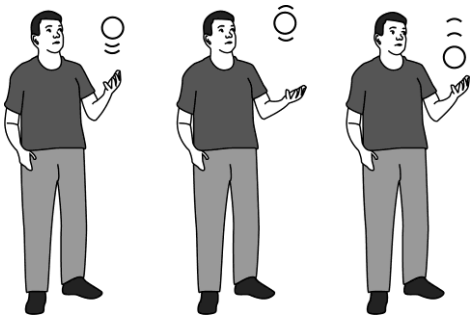
NOTES

When only conservative forces act on an object, kinetic energy is converted to potential energy or vice versa. If an object is thrown into the air, the object has kinetic energy (energy of motion) as it leaves the hand. Eventually, this object will reach its highest point where it will come to a stop. At this point it has potential energy. The kinetic energy was changed to potential energy, and as the object returns to the hand, the potential is converted back to kinetic. However, the sum of the kinetic and potential energies is constant throughout this motion. The mechanical energy of the system remains constant.



The law of conservation of mechanical energy states that in a frictionless system mechanical energy is conserved. The system must be frictionless because the force of friction is a non-conservative force.

This law can be explained mathematically as:

$$\Delta E_k + \Delta E_p = 0$$

$$\Delta E_k = -\Delta E_p$$

$$\frac{1}{2}m\left(v_f^2 - v_0^2\right) = -mg\Delta h$$

$$\frac{1}{2}m = -mg\left(h_f - h_0\right)$$

$\therefore$ the gain in $E_k =$ the loss in E_p

NOTE: Δh may be negative.

If there is friction, then some mechanical energy is converted to thermal energy. The law of conservation of mechanical energy no longer applies because the energy of the system is not conserved.

NOTES

Example

A heavy object is dropped from a vertical distance of 12.0 m above the ground. What is the speed of the object as it hits the ground?

Solution

$$\Delta E_k + \Delta E_p = 0$$
$$\Delta E_k = -\Delta E_p$$
$$\frac{1}{2}m\left(v_f^{\,2} - v_0^{\,2}\right) = -mg\Delta h$$
$$\frac{1}{2} = -g\Delta h$$
$$\frac{1}{2}\left(v_f^{\,2} - 0\right) = -\left(9.81\ \text{m/s}^2\right)\left(-12.0\ \text{m}\right)$$
$$v_f = \sqrt{-\left(9.81\ \text{m/s}^2\right)\left(-12.0\ \text{m}\right)(2)}$$
$$= 15.3\ \text{m/s}$$

Example

A heavy object is thrown vertically down from the top of a 1.00×10^2 m building at a velocity of 10.0 m/s. What is the speed as it reaches the ground?

Solution

$$\Delta E_k + \Delta E_p = 0$$
$$\Delta E_k = -\Delta E_p$$
$$\frac{1}{2}m\left(v_f^{\,2} - v_0^{\,2}\right) = -mg\Delta h$$
$$\frac{1}{2}\left(v_f^{\,2} - v_0^{\,2}\right) = -g\Delta h$$
$$\frac{1}{2}\left(v_f^{\,2} - \left(10.0\ \text{m/s}\right)^2\right) = -\left(9.81\ \text{m/s}^2\right)\left(-1.00 \times 10^2\ \text{m}\right)$$
$$v_f = \sqrt{-\left(9.81\ \text{m/s}^2\right)\left(-1.00 \times 10^2\ \text{m}\right)(2) + \left(10.0\ \text{m/s}\right)^2}$$
$$= 45.4\ \text{m/s}$$

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PRACTICE EXERCISES

Formulas: $\Delta E_k + \Delta E_p = 0$ $E_p = mgh$ $E_k = \frac{1}{2}mv^2$

Although there are other methods to solve these problems, use the law of conservation of mechanical energy.

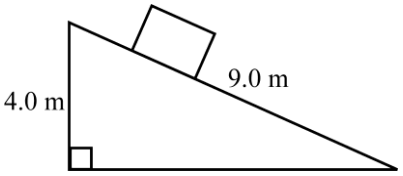
1. A heavy object is dropped. If this object reaches the floor at a speed of 3.2 m/s, from what height was it dropped?

2. A heavy object is dropped from a vertical height of 8.0 m above the ground. What is the speed of this object as it hits the ground?

3. A heavy object is dropped from the top of a building. If this object hits the ground with a speed of 37.0 m/s, how tall was the building?

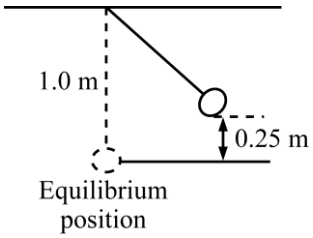
4. A heavy object is thrown vertically down from the top of a 1.3×10^2 m building at a speed of 11.0 m/s. What is the velocity as it hits the ground?

5.



A heavy box slides down a frictionless incline as shown in the diagram. If the box starts from rest at the top of the incline, what is its speed at the bottom?

6.



A pendulum is dropped from the position as shown in the diagram 0.25 m above the equilibrium position. What is the speed of the pendulum bob as it passes through the equilibrium position?

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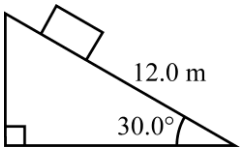
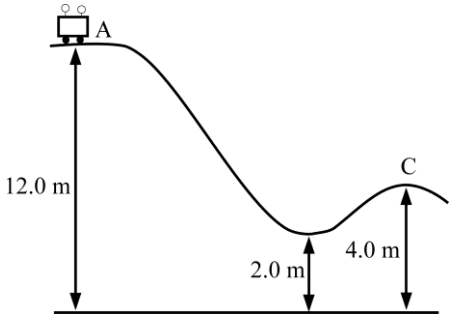
7.



A heavy box slides down a frictionless incline as shown in the diagram. If the box starts from rest at the top of the incline, what is its speed at the bottom?

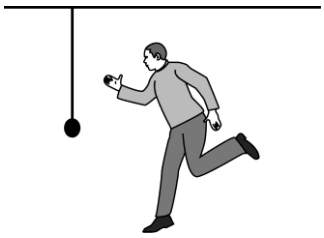
8.



A roller coaster car starts from rest at point A. What is the speed of this car at point C if the track is frictionless?

9. A 2.5 kg object is dropped from a height of 10.0 m above the ground. Calculate the speed of the object as it hits the ground.

10.



A student is running at a speed of 3.5 m/s and grabs a long rope that is hanging vertically from a tree. How high can the student swing?

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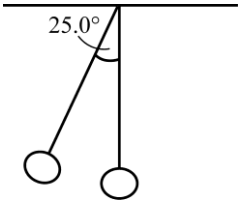
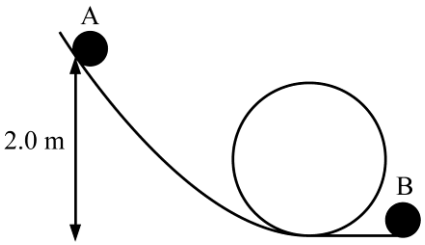
11. 

A pendulum is 1.20 m long. What is the speed of the pendulum bob when it passes through its equilibrium position if it is pulled aside until it makes a 25.0° angle to the vertical and then released?

HINT: Is it possible to determine the vertical drop of the pendulum bob and then use the law of conservation of mechanical energy?

12. A 45 kg student runs with a speed of 2.0 m/s off a horizontal ledge that is 5.0 m above a swimming pool. Calculate the vertical speed of the student when she hits the water.

13.



A 1.0 kg box slides without friction around the loop-the-loop apparatus as shown in the diagram. If the object starts from rest at point A, and the radius of the loop is 0.75 m, what is the speed of the box at point B?

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Lesson 5 POWER

NOTES

Power is the rate at which work is done. It can also be thought of as the rate at which the energy of the system changes. Because of this, power is often confused with work. You will note that in the previous discussions of work, time was not a concern. Power is related with time because power is the *rate* of doing work.

Power is defined as the rate of doing work.

The power, *P*, to do work over a period of time, *t*, is mathematically defined as:

$$P = \frac{W}{t} \text{ and since } W = \Delta E$$
$$P = \frac{\Delta E}{t}$$

$$P = \frac{W}{t} \text{ or } P = \frac{\Delta E}{t}$$

Work and time are both scalar quantities and therefore, power is also a scalar quantity. Since work and energy are measured in joules and time is measured in seconds, power is measured in joules per second (J/s) or watts (W). Try not to confuse the symbol for watts (W) with the symbol for work (*W)*, as it is a common mistake.

$P = Fv$

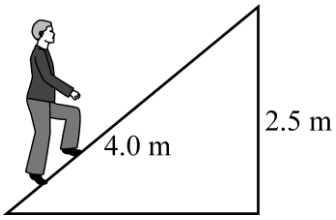
A second useful equation for power can be derived using the definition of work and speed:

$$P = \frac{W}{t} \text{ subbing in } W = Fd$$
$$P = \frac{Fd}{t} \text{ subbing in } v = \frac{d}{t}$$
$$P = Fv$$

This equation shows that the product of the magnitude of the force acting on an object with the speed of the object gives the power.

NOTES

Example



A 60.0 kg student runs at a constant velocity up the incline described in the diagram in 4.5 s. Calculate the power output of the student.

Solution

Find the work first. Since the work is against gravity, use the formula:

$$W = mgh$$
$$= (60.0\text{ kg})(9.80\text{ m/s}^2)(2.5\text{ m})$$
$$= 1.47\times10^3\text{ J}$$
$$P = \frac{W}{t}$$
$$= \frac{1.47\times10^3\text{ J}}{4.5\text{ s}}$$
$$= 3.3\times10^2\text{ W}$$

Efficiency

$$= \frac{\text{work out}}{\text{work in}}\times100\%$$

or

$$= \frac{\text{power out}}{\text{power in}}\times100\%$$

Example

A 1.00×10^3 kg car accelerates from rest to a velocity of 15.0 m/s in 4.00 s. Calculate the power output of the car in this 4.00 s.

Solution

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_0}{t}$$
$$= \frac{15.0\text{ m/s}}{4.00\text{ s}}$$
$$= 3.750\text{ m/s}^2$$

$$\vec{F}_{net} = m\vec{a}$$
$$= (1.00\times10^3\text{ kg})(3.75\text{ m/s}^2)$$
$$= 3.750\times10^3\text{ N}$$

$$\vec{d} = \left(\frac{\vec{v}_f + \vec{v}_0}{2}\right)t$$
$$= \left(\frac{15.0\text{ m/s}}{2}\right)4.00\text{ s}$$
$$= 30.00\text{ m}$$

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NOTES

$$W = Fd$$
$$= (3.75 \times 10^3 \text{ N})(30.0 \text{ m})$$
$$= 1.125 \times 10^5 \text{ J}$$

$$P = \frac{W}{t}$$
$$= \frac{1.13 \times 10^5 \text{ J}}{4.00 \text{ s}}$$
$$= 2.81 \times 10^4 \text{ W}$$

EFFICIENCY

Efficiency is the ratio of the useful work output by a system to the work input into a system. Mathematically, this is defined as:

$$\text{efficiency} = \frac{\text{work out}}{\text{work in}}$$

or

$$\text{efficiency} = \frac{\text{power out}}{\text{power in}}$$

Efficiency is often expressed as a percent. This is expressed as:

$$\text{efficiency} = \frac{\text{work out}}{\text{work in}} \times 100\%$$

or

$$\text{efficiency} = \frac{\text{power out}}{\text{power in}} \times 100\%$$

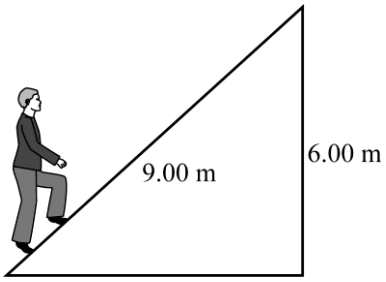
It is helpful to remember that the power or work input into the system is always greater than the power or work output from the system, that is:

$$\text{power in} > \text{power out}$$
$$\text{work in} > \text{work out}$$

PRACTICE EXERCISES

Formulas: $P = \dfrac{W}{t}$ $P = Fv$

1.



A 45.0 kg student runs at a constant velocity up the incline described in the diagram. If the power output of the student is 1.50×10^3 W, how long does it take the student to run the 9.0 m along the incline?

2. A 20.0 kg object is lifted vertically at a constant velocity 2.50 m in 2.00 s by a student. Calculate the power output of the student.

3. A 2.00 kg object is accelerated uniformly from rest to 3.00 m/s while moving 1.5 m across a level frictionless surface. Calculate the power output.

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4.

An 8.5×10^2 kg elevator (including occupants) is pulled up at a constant velocity of 1.00 m/s by a 10.0 kW electric motor. Calculate the efficiency of the electric motor.

5. A 5.0 kg object is accelerated uniformly from rest to 6.0 m/s by an applied force while moving 2.0 m across a level surface. If the force of friction is 4.0 N, calculate the power output.

6. A 5.00×10^2 W electric motor lifts a 20.0 kg object 5.00 m in 3.50 s. What is the efficiency of the motor?

7. If a 1.00 kW electric motor has an efficiency of 82%, how long will it take to lift a 50.0 kg object to a height of 8.00 m?

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Lesson 6 MACHINES AND EFFICIENCY

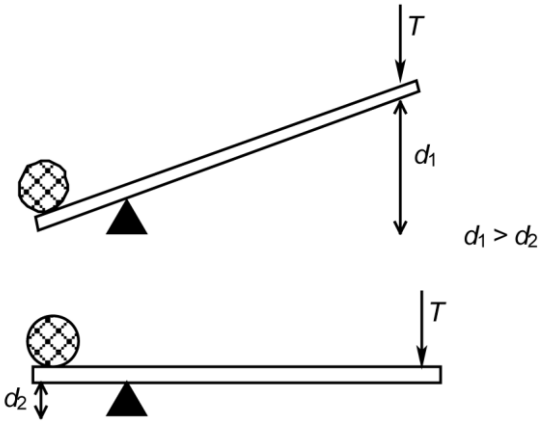
NOTES

Electric appliances are devices that convert electric energy into other forms of energy. For example, an incandescent light bulb is designed to convert electric energy to light. However, it is only 5% efficient in doing this. Other appliances also have a certain efficiency in converting electric energy to the desired energy. An old gas furnace may only be 60% efficient in converting the chemical energy in the gas to useful thermal energy that heats our homes.

The efficiency of machines can also be measured. Machines can be simple like levers, pulleys and inclined planes; or they can be complex like bicycles and cars. Complex machines are combinations of simple machines. Machines are devices that are used to do work. They are often used to reduce the force that needs to be exerted on an object.

Remember:
$\text{work} = \text{force} \times \text{displacement}$ or $W = Fd$

If an object is lifted vertically 10.0 cm by using a lever, the same work is done on the object as lifting it vertically 10.0 cm without the lever. However, the lever allows for the exertion of less force. This means that the force must be exerted through a greater distance.



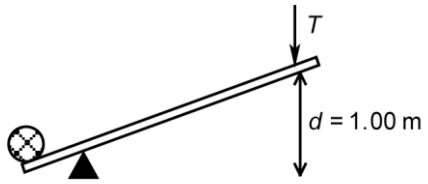
Similar to a light bulb or a gas furnace, when using machines, some of the energy used is wasted. Most of this energy turns into heat through the force of friction. A machine always has an efficiency that is less than 100%.

NOTES

Example

A lever is used to lift a 50.0 kg object 10.0 cm. This is done by applying a force of 75.0 N to the end of the lever as shown in the diagram. The displacement of the end of the lever is 1.0 m. What is the efficiency of this simple machine?

Solution



$$\begin{aligned}\text{work in} &= Fd \\ &= (75.0\ \text{N})(1.00\ \text{m}) \\ &= 75.0\ \text{J}\end{aligned}$$

$$\begin{aligned}\text{work out} &= mgh \\ &= (50.0\ \text{kg})(9.81\ \text{m/s}^2)(0.100\ \text{m}) \\ &= 49.05\ \text{J}\end{aligned}$$

$$\begin{aligned}\text{efficiency} &= \frac{\text{work out}}{\text{work in}} \times 100\% \\ &= \frac{49.05\ \text{J}}{75.0\ \text{J}} \times 100\% \\ &= 65.4\%\end{aligned}$$

Example 2

A pulley system is 75% efficient in lifting a 925 N object 7.0 m. What is the work input?

Solution

$$\text{efficiency} = \frac{\text{work out}}{\text{work in}} \times 100\%$$

$$\begin{aligned}\text{work in} &= \frac{\text{work out}}{\text{efficiency}} \times 100\% \\ &= \frac{(925\ \text{N})(7.0\ \text{m})}{75\%} \times 100\% \\ &= \frac{(925\ \text{N})(7.0\ \text{m})}{0.75} \times 1.00 \\ &= 8.6 \times 10^3\ \text{J}\end{aligned}$$

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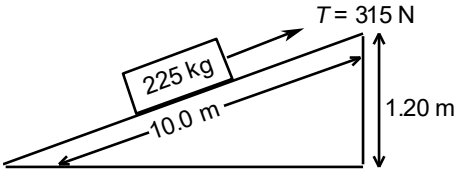
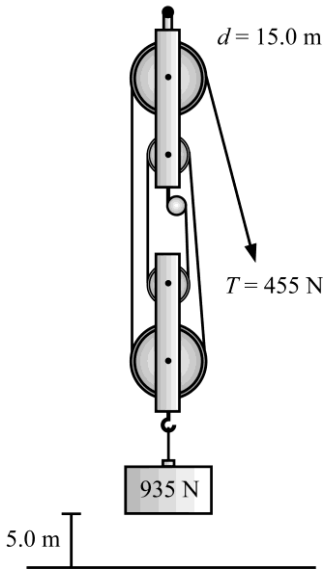
PRACTICE EXERCISES

Formulas: $\text{efficiency} = \dfrac{\text{work out}}{\text{work in}} \times 100\%$ $\text{efficiency} = \dfrac{\text{power out}}{\text{power in}} \times 100\%$

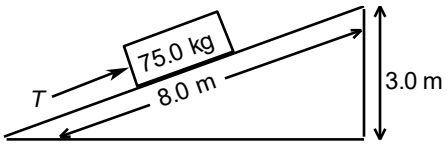
1. A 225 kg box is loaded onto the back of a truck by pulling it up an inclined plane by a force of 315 N parallel to the incline as shown in the diagram. If the bed of the truck is 1.20 m above the ground, what is the efficiency of this simple machine in loading the box into the truck?



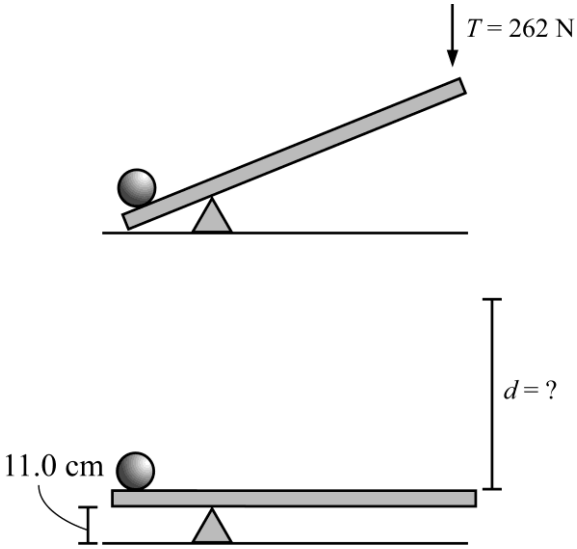
2. You exert a force of 455 N in pulling 15.0 m of rope using a pulley system to lift a 935 N object 5.0 m. What is the efficient of the pulley system?



3. What force is needed to push a 75.0 kg box up the incline described in the diagram if the efficiency of the machine (ramp) is 78.5%?



4. It is estimated that the lever system shown in the diagram is 63% efficient. You use this system to lift a 95 kg object 11.0 cm by exerting a force of 262 N. What is the displacement of the right end of the lever?



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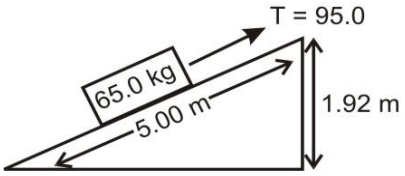
5. A 65.0 kg object is pulled at a constant velocity up an incline with a force of 359 N parallel to the incline as shown in the diagram.



Assuming that the difference between the work output and input is the work required to overcome friction, what is the force of friction along the incline (the length of the incline is 5.00 m, the height of the incline is 1.92 m and the efficiency of this simple machine is 68.2%)?

6. A 1.00×10^4 W electric motor is used to lift an elevator and occupants (mass = 1.30×10^3 kg) to a height of 25.0 m in 60.0 s at a constant velocity. What is the efficiency of this motor?

Lesson 7 ELECTRIC CURRENT

NOTES

ELECTRICITY

The introduction of the battery initiated new investigations in electricity: investigations that no longer involved only static charges or the discharge of these static charges. Investigations now could involve electric currents (flow of a charge through conductors).

Electric current is the rate of flow of electrical charges.

Electric current is the rate of flow of electrical charge. In metallic conductors, it is the electrons that flow. The symbol of electric current is *I*.

Electric current can be described using:
conventional current
(positive flow)
electron flow

Definition: rate of flow of electrical charge $\left(I = \frac{q}{t} \right)$

Units: $\frac{\text{C}}{\text{s}}$ (which is called the ampere—A).

1 ampere = $\frac{1 \text{ coulomb}}{\text{s}} = \frac{\text{C}}{\text{s}}$

Charges will not move through a conductor unless there is a potential difference (voltage). In an electric circuit, there must be a voltage source and a complete pathway for the charge to flow through.

Batteries in an electric circuit are voltage sources. Power companies use electric generators as voltage sources. (We will discuss generators later.) The current through a conductor depends on the potential difference and the resistance in the conductor. Current (rate of flow) will vary directly with the potential difference. It's like the current in a river. The greater the slope of the river bed, the faster the water flows. In the same way, the greater the potential difference (voltage), the greater the rate of flow (current).

$I \propto V$

Resistance in a conductor produces heat.

There is always resistance to the flow (movement) of charged particles through a conductor. This is why conductors get warm or hot when a current is flowing through them.

Current varies inversely with resistance.

$I \propto \frac{1}{R}$

We use *R* as the symbol for resistance.

The relationship $I \propto V$ and $I \propto \frac{1}{R}$ or $I \propto \frac{V}{R}$ was discovered by Ohm in 1826 and became known as Ohm's Law.

The above relationship is generally written as $I = \frac{V}{R}$

Ohm's Law: $V = IR$

Ohm's Law is usually written as $V = IR$.

The unit of resistance is called the ohm (symbol Ω).

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ELECTRIC CURRENT

NOTES

There are two ways of thinking about electric current.

- Electron flow—it is the electrons that flow through a conductor when there is an electric current. Electrons flow from the negative terminal to the positive terminal.
- Conventional current—is positive charge flow. Positive charges flow from the positive terminal to the negative terminal.

Conventional current is positive charge flow.

We know that it is actually the electrons that flow through a conductor; so why do we talk about conventional current? Positive charge flow has a historical basis; but also it probably goes back to our definition for the direction of an electric field and electric potential. Electric potential is defined in terms of a moving positive charge. Remember that the direction of an electric field is defined as the direction of the force on a positive test object.

Most college and university physics texts will use conventional current to describe electric current rather than electron flow. On your provincial exam, *electric current* refers to conventional current. And unless we indicate otherwise in this book, electric current will be conventional current.

ELECTRIC POWER

Power is the rate of doing work.

Power is the rate of doing work or using energy.

$$P = \frac{W}{t} = \frac{\Delta E}{t}$$

ΔE = energy transferred
W = work done
P = power

The unit of power is the watt (W).

From this definition of power and Ohm's Law, we can derive other relationships for power using electrical terms.

$$P = IV$$
$$P = I^2R$$
$$P = \frac{V^2}{R}$$

Derivation:
P = power, I = current, V = potential difference, and R = resistance.

Using $P = \frac{W}{t}$ and recalling that $V = \frac{\Delta E}{q}$ or $\Delta E = Vq$, then $\Delta E =$ work or change in energy.

NOTES

Therefore, $P = \frac{Vq}{t}$ and $I = \frac{q}{t}$

Hence, $P = IV$.

Again $I = \frac{V}{R}$. Therefore, $P = \frac{V}{R}V$ or $P = \frac{V^2}{R}$

If $I = \frac{V}{R}$, then $V = IR$.

Therefore, $P = I(IR)$, or $P = I^2R$

Example

What is the electric current through a conductor if a charge of 2.0 C flows through a point in the conductor in 10 s?

Solution

$$I = \frac{q}{t}$$
$$= \frac{2.0\text{ C}}{10\text{ s}}$$
$$= 2.0 \times 10^{-1}\text{ A}$$

Example

Calculate the resistance in a conductor if the potential difference is 6.0 V and the current is 10 A.

Solution

$$V = IR$$
$$R = \frac{V}{I}$$
$$= \frac{6.0\text{ V}}{10\text{ A}}$$
$$= 6.0 \times 10^{-1}\ \Omega$$

Example

An electrical appliance uses 1.00×10^2 W when connected to a 1.20×10^2 V power line. What is the resistance in the appliance?

Solution

$$P = \frac{V^2}{R}$$
$$R = \frac{V^2}{P}$$
$$= \frac{(1.20 \times 10^2\text{ V})^2}{1.00 \times 10^2\text{ W}}$$
$$= 1.44 \times 10^2\ \Omega$$

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NOTES

EFFICIENCY

Efficiency is defined as the ratio of the useful power output to the total power input. This can be expressed as follows:

$$\text{efficiency} = \frac{\text{power out}}{\text{power in}} \times 100\%$$

This can equivalently be stated as:

$$\text{efficiency} = \frac{\text{work out}}{\text{work in}} \times 100\%$$

Example

A light bulb is supplied 80.0 W of power. The bulb emits 56.0 J each second as light energy. What is the efficiency of the light bulb?

Solution

$$\text{power out} = 56.0 \text{ J/s} = 56.0 \text{ W}$$
$$\text{power in} = 80.0 \text{ W}$$
$$\text{efficiency} = \frac{\text{power out}}{\text{power in}} \times 100\%$$
$$= \frac{56.0 \text{ W}}{80.0 \text{ W}} \times 100\%$$
$$= 70.0\%$$

Consider the equation $P = IV$. This equation states that the amount of power is equal to the current (I) times the voltage (V). If the current is lowered and the voltage is increased, the same amount of power (P) can be generated.

Now, consider the equation $P = I^2R$. This equation can be used to calculate the power loss due to resistance in the cables used to transmit power, and it implies that the power lost to heat in transmission lines can be reduced if the transmission current is decreased. Note that in the equation, current (I) is squared. If the current was reduced by a factor of 100 times, the power loss would be 100^2 or 10 000 times less. To receive the same amount of power in people's homes, the voltage must be increased to compensate for the reduction in current.

PRACTICE EXERCISES

Formulas: $I = \frac{q}{t}$ $V = IR$ $P = IV$ $P = I^2R$ $P = \frac{V^2}{R}$

1. A current of 3.60 A flows for 15.3 s through a conductor. Calculate the number of electrons that pass through a point of the conductor during this time.

2. How long would it take 2.0×10^{20} electrons to pass through a point in a conductor if the current was 10.0 A?

3. Calculate the current through a conductor if a charge of 5.60 C passes through a point in the conductor in 15.4 s.

4. What potential difference is required across a conductor to produce a current of 8.00 A if there is a resistance in the conductor of 12.0 Ω?

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5. What is the heat produced in a conductor in 25.0 s if there is a current of 11.0 A and a resistance in the conductor of 7.20 Ω?

6. A particular conductor produces 1.50×10^2 J of heat in 5.50 s. If the current through the conductor is 10.0 A, what is the resistance of the conductor?

7. What is the current through a 4.00×10^2 W electric appliance when it is connected to a 1.20×10^2 V power line?

8. **a)** When an electric appliance is connected to a 1.20×10^2 V power line, there is a current through the appliance of 18.3 A. What is the resistance of the appliance?

b) What is the average amount of energy given to each electron by the power line?

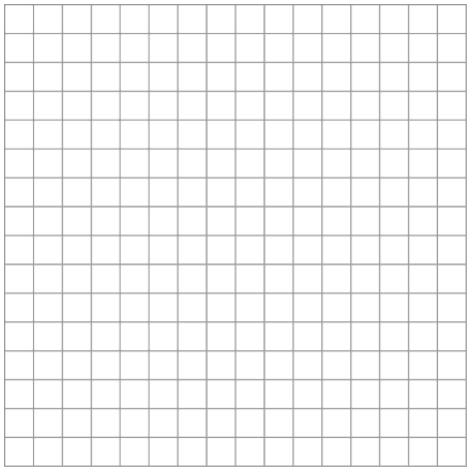
9. **a)** What potential difference is required across an electrical appliance to produce a current of 20.0 A when there is a resistance in the appliance of 6.00 Ω?

b) How many electrons pass through the electrical appliance every minute?

10. A student designed an experiment in order to measure the current through a resistor when manipulating the potential difference. The following data were recorded:

Potential difference (V)	**Current (A)**
3.00	0.151
6.00	0.310
9.00	0.448
12.00	0.611
15.00	0.750

a) Using the data above, draw a graph showing the relationship between the current and the potential difference.



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b) Using only the graph, find the resistance of the resistor.

11. If electricity cost \$0.060 per kilowatt hour, what does it cost to operate a 1.0×10^3 W appliance for 5.0 h?

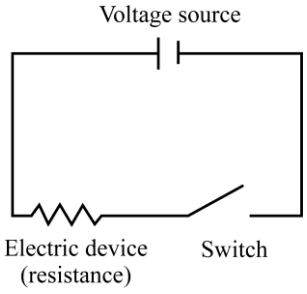
12. A student forgets to turn off a 6.00×10^2 W block heater of a car when the weather turns warm. If 14 h goes by before he shuts it off, how much energy is used by the heater?

13. A 45 kg object is lifted vertically at a constant speed to a height of 9.0 m by a 7.5×10^2 W electric motor. If this motor is 25% efficient in converting electric energy to mechanical energy, how long does the motor take to lift the object?

Lesson 8 ELECTRIC CIRCUITS

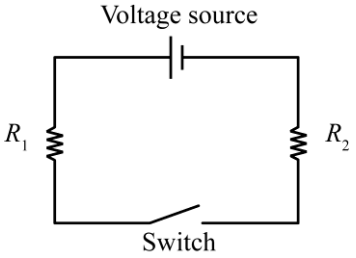
NOTES

An electric circuit is a pathway that allows charges to flow. In an electric circuit, there must be a voltage source—for example, a battery. The voltage source (battery) provides the electrical energy. An electric device (light bulb, toaster, radio, door bell, etc.) is placed in this circuit so that the device becomes part of the circuit. That is, the charge flows through the device, and as the charge flows, the electrical energy is converted to other forms of energy—one of these forms is heat. The reason heat is produced is that the electric device provides some resistance to the charge flow. There is usually a switch in an electric circuit. A switch is a means of breaking the pathway (circuit). If the pathway is broken, the charge cannot flow. Below is a diagram of a simple electric circuit showing the symbols for the voltage source, the electric device (resistance), and the switch.



The circuits in our homes, cars, etc., have circuit breakers or fuses. These act like switches, preventing the charge from flowing through the circuit by creating a gap (break) in the pathway. Circuit breakers and fuses are heat sensitive. As the current through a conductor increases, so does the temperature of the conductor (circuit). If the current is too great, the heat produced will melt the fuse or trip the breaker.

A circuit can be described as a series or parallel circuit.



The circuit above is called a series circuit because the same amount of electrons must pass through both resistors. It is like running hurdles. You jump one hurdle, then you must jump the next. You run hurdles in series. Some Christmas lights are in series—when one light goes out, they all go out. If you look at the diagram, you can see that if one resistor burns out (like a light bulb) there is no longer a path for the electrons to flow through. Therefore, there will be no charge passing through R_2.

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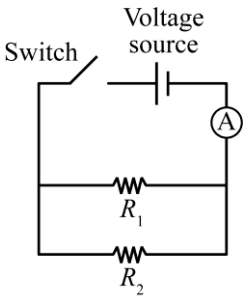
Parallel Circuit



NOTES

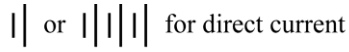
Parallel Circuit: resistors are in parallel

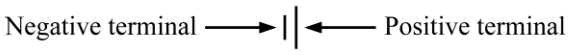
The circuit above is called a parallel circuit because the charge has more than one path. The charge can travel through R_1 or R_2. If R_1 burns out, the charge can still travel through R_2. This is the type of electrical circuit in our homes.

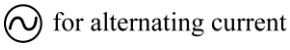
DIRECT AND ALTERNATING CURRENT

Electric current can be direct (DC) or alternating (AC). Batteries produce direct current; that is, the conventional current always flows in the same direction through the circuit from the positive terminal to the negative terminal . Electrical generators can produce either direct current or alternating current. With alternating current, the conventional current in the circuit flows one direction and then the other—the current alternates in one direction and then the other. The circuits in our homes use alternating current. In this unit, we will study direct current (alternating current in the next unit). Many of the principles of direct current also apply to alternating current.

When drawing circuits we use the symbols:







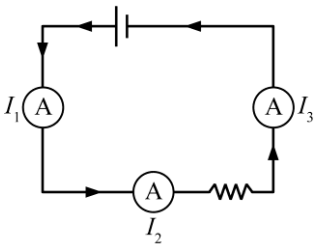
At this point, we will introduce Kirchhoff's current and voltage laws. The laws will apply to all DC circuits (both series and parallel).

NOTES

KIRCHHOFF'S CURRENT LAW

Kirchhoff's Current and Voltage Laws apply to all DC circuits.

Series Circuit

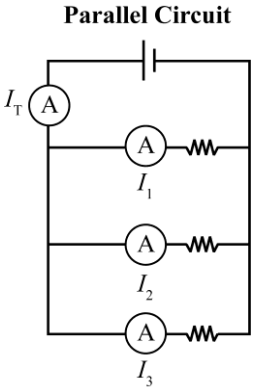


In a series circuit, because there is only one path, the current must be the same at all points.

$I_T = I_1 = I_2 = I_3$

Think of the charge as soldiers marching. The number of soldiers marching at any point must be the same.

Parallel Circuit



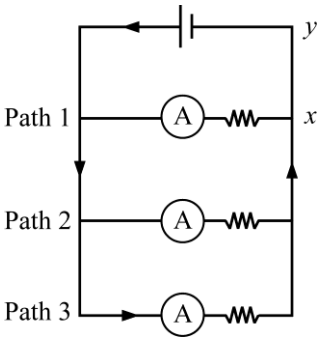
In a parallel circuit, the charge (soldiers) can take different paths. Therefore, the amount of charge (soldiers) at any point can be different.

$I_T = I_1 + I_2 + I_3$

Kirchhoff's Current Law is just a form of the Law of Conservation of Electric Charge.

Kirchhoff's law is stated as: The sum of the currents entering a junction equals the sum of the currents leaving the junction.

Remember, current is the movement of electrons, and if electrons move into a path, they must come out. In a way, this is just a different expression of the Law of Conservation of Electric Charge.



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NOTES

Again, if we use the soldier analogy, the soldiers that take path 1 and those that take path 2 and path 3 will come together at *x*. The soldiers marching from *x* to *y* will equal the sum of the soldiers taking path 1,path 2 and those taking path 3.

That is:

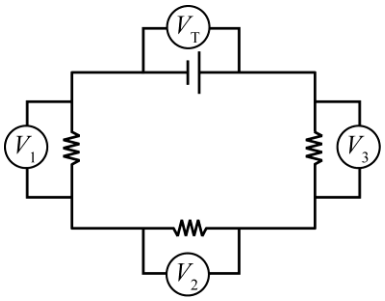
$I_T = I_1 + I_2 + I_3 + \text{etc}$

Kirchhoff's Voltage Law is a form of the Law of Conservation of Energy.

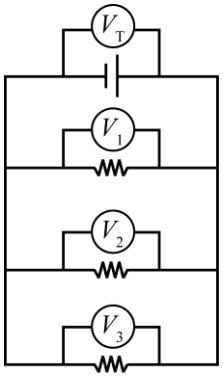
KIRCHHOFF'S VOLTAGE LAW

Kirchhoff's Voltage Law is stated as: The sum of the potential differences in any circuit must equal zero.

This is just a way of expressing the Law of Conservation of Energy. Remember, there is an increase in the potential across the terminals of a battery or power source. Also, remember that there is a potential loss (drop) across a resistor. Kirchhoff's Voltage Law is saying that these increases and losses must add up to zero in any circuit.



$V_T = V_1 + V_2 + V_3$



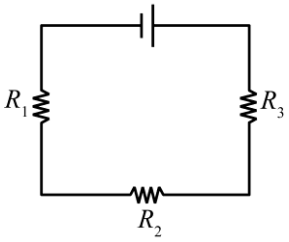
$V_T = V_1 = V_2 = V_3$

Note that in a parallel circuit, the potential difference is the same across all resistors in the circuit.

NOTES

Kirchhoff's Laws and Ohm's Law can be used to derive equations for finding the total (equivalent) resistance in DC circuits. However, we will not derive them, but let's try to reason out these equations.

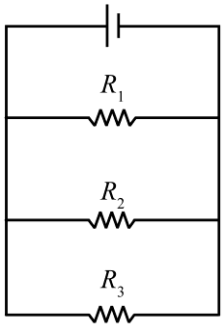
Series circuit



$$R_{eq} = R_1 + R_2 + R_3 .$$

The total resistance in a series circuit is the sum of all the resistances in the circuit. Let's again use the analogy of the marching soldiers. The charge (soldiers) must pass through all resistance. The total resistance it passes through is the sum of them all.

Parallel Circuit



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Note that in a parallel circuit, the more paths there are, the less the total resistance will be. If all the charge (soldiers) has to pass through one resistor (path), a lot of friction (resistance) will be created. However, if the charge (soldiers) has a number of paths, there is more room and, therefore, less resistance.

MATHEMATICS OF CIRCUITS

Many electric circuits are a combination of series and parallel circuits or connections. Therefore we will need to be able to solve problems involving:

- Series circuits
- Parallel circuits
- Combination circuits

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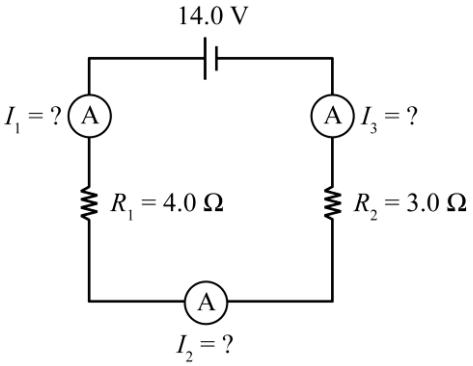
NOTES

Example



What are the values of I_1, I_2, and I_3 in this circuit?

Solution

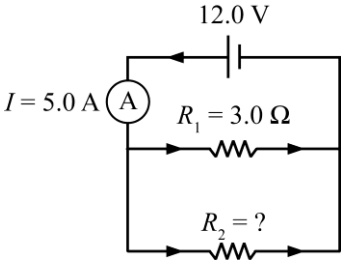
Ohm's Law: $V_T = I_T R_T$

$$R_T = R_{eq} = R_1 + R_2$$
$$= 4.0\ \Omega + 3.0\ \Omega$$
$$= 7.0\ \Omega$$
$$\therefore I_T = \frac{V_T}{R_T}$$
$$= \frac{14.0\ \text{V}}{7.0\ \Omega}$$
$$= 2.0\ \text{A}$$

In a series circuit:

$$I = I_1 = I_2 = I_3 \ldots$$
$$I_1 = I_2 = I_3 = 2.0\ \text{A}$$

Example



What is the value of R_2 in this circuit?

Solution

Ohm's Law $V = IR$

$$R_{eq} = R = \frac{V}{I}$$
$$= \frac{12.0\ \text{V}}{5.0\ \text{A}}$$
$$= 2.4\ \Omega$$

NOTES

In a parallel circuit:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

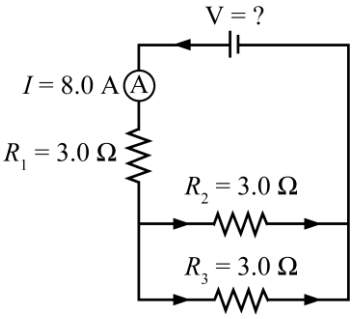
$$\frac{1}{2.4\ \Omega} = \frac{1}{3.0\ \Omega} + \frac{1}{R_2}$$

$$\frac{1}{R_2} = \frac{1}{2.4\ \Omega} - \frac{1}{3.0\ \Omega}$$

$$= \frac{3.0 - 2.4}{7.2\ \Omega}$$

$$= 12\ \Omega$$

Example



What is the potential difference supplied by the power source in this circuit?

Solution

Find equivalent resistance in the two parallel resistors first:

$$\frac{1}{R_{eq(p)}} = \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{3.0\ \Omega} + \frac{1}{3.0\ \Omega}$$

$$= \frac{1.0 + 1.0}{3.0\ \Omega}$$

$$= 1.5\ \Omega$$

Now, add in the series resistors:

$$R_{eq(s)} = R_{eq(p)} + R_1$$

$$= 1.5\ \Omega + 3.0\ \Omega$$

$$= 4.5\ \Omega$$

Ohm's Law

$$V = IR = IR_{eq(s)}$$

$$= (8.0\ \text{A})(4.5\ \Omega)$$

$$= 36\ \text{V}$$

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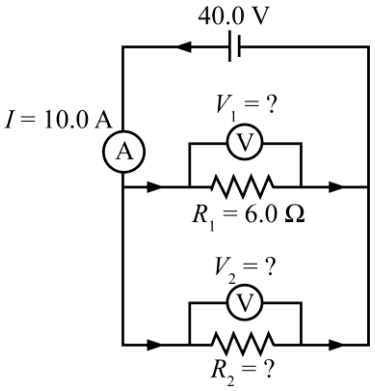
NOTES

Example



What are the values of V_1, V_2, and R_2 in this circuit?

Solution

For a parallel circuit:

$V = V_1 = V_2 \ldots$

As $V = 40.0$ V

$V_1 = 40.0$ V

$V_2 = 40.0$ V

Ohm's Law

$V = IR$

$$R_{eq} = R = \frac{V}{I}$$
$$= \frac{40.0 \text{ V}}{10.0 \text{ A}}$$
$$= 4.0\ \Omega$$

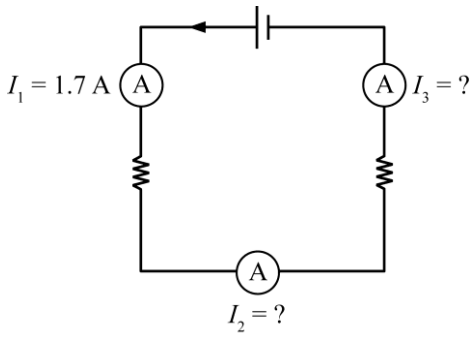
For a parallel circuit:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$\frac{1}{4.0\ \Omega} = \frac{1}{6.0\ \Omega} + \frac{1}{R_2}$$
$$\frac{1}{R_2} = \frac{1}{4.0\ \Omega} - \frac{1}{6.0\ \Omega}$$
$$= \frac{3.0 - 2.0}{12\ \Omega}$$
$$R_2 = 12\ \Omega$$

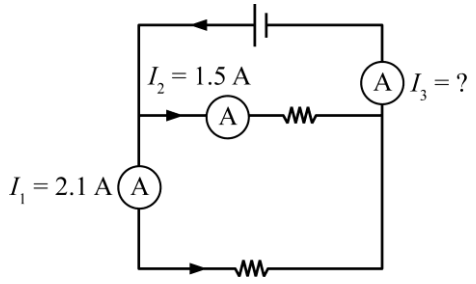
PRACTICE EXERCISES

Formulas:	For series circuits	For parallel circuits
	$I_T = I_1 = I_2 = I_3 = ...$	$I_T = I_1 + I_2 + I_3 + \ \ldots$
$V = IR$	$V_T = V_1 + \ V_2 + V_3 + \ldots$	$V_T = V_1 = V_2 = V_3 = \ \ldots$
	$R_{eq} = R_1 + R_2 + R_3 + \ \ldots$	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$

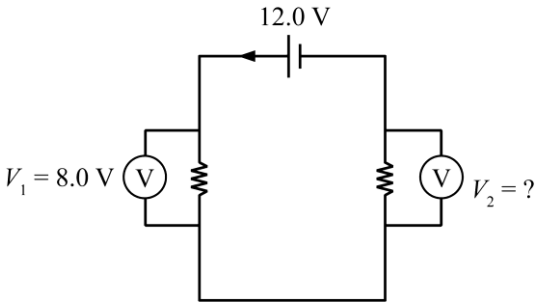
1. What are the values for I_2 and I_3 in the circuit?



2. What is the value of I_3 in the circuit?



3. What is the value of V_2 in the circuit?



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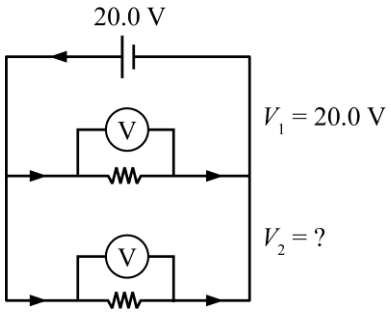
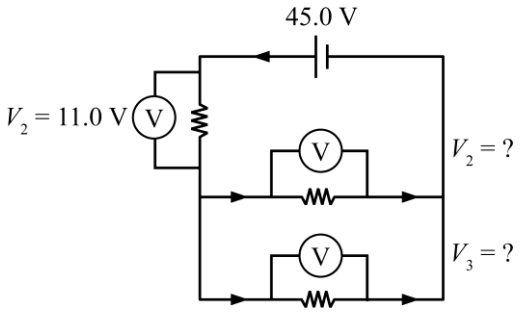
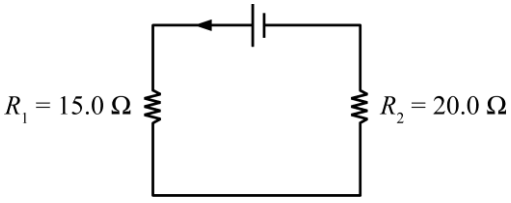
4. What is the value of V_2 in the circuit?



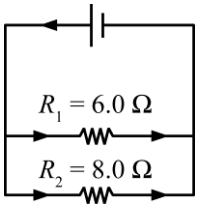
5. What are the values of V_2 and V_3 in the circuit?



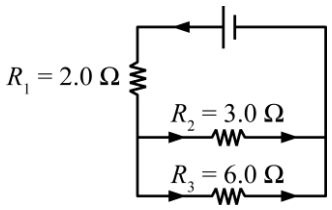
6. What is the total resistance in the circuit?



7. What is the total resistance in the circuit?



8. What is the total resistance in the circuit?



9. What is the total resistance in a circuit containing three resistors in series? The values of these resistors are 9.0 Ω, 3.0 Ω and 12.0 Ω.

10. What is the total resistance in a circuit containing three resistors in parallel? The values of these resistors are 2.0 Ω, 4.0 Ω and 8.0 Ω.

11. The total resistance in a circuit containing three resistors in parallel is 2.0 Ω. If the values of two of these resistors are 4.5 Ω and 9.0 Ω, what is the value of the third resistor?

12. The total resistance in a circuit containing three resistors in series is 12.0 Ω. If the values of two of these resistors are 6.0 Ω and 4.0 Ω, what is the value of the third resistor?

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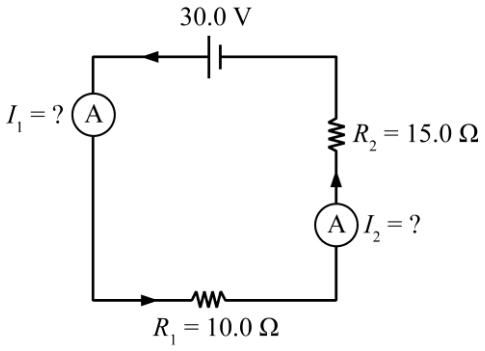
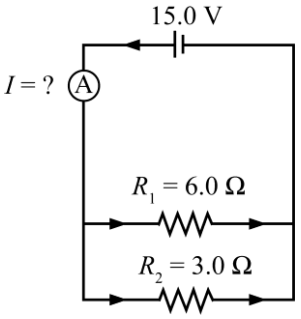
13. a) What are the values of I_1 and I_2 in the circuit?



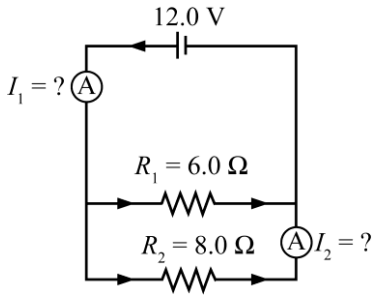
b) What is the power dissipated in R_1?

14. a) What is the value of *I* in the circuit?



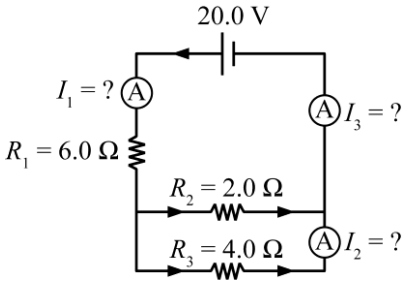
b) What is the power dissipated in R_1?

15. a) What are the values of I_1 and I_2 in the circuit?



b) What is the total power dissipated in the circuit?

16. a) What are the values of I_1, I_2 and I_3 in this circuit?



b) What is the total power dissipated in the circuit?

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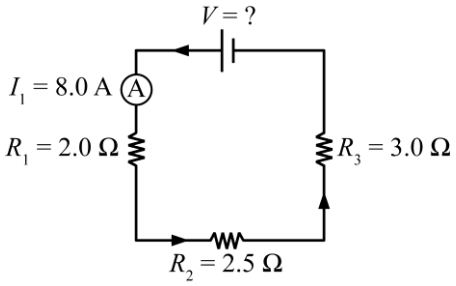
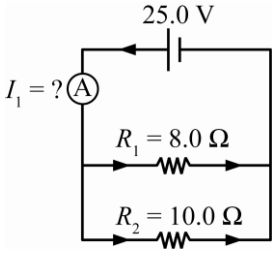
17. a) What is the potential difference supplied by the power source in the circuit?



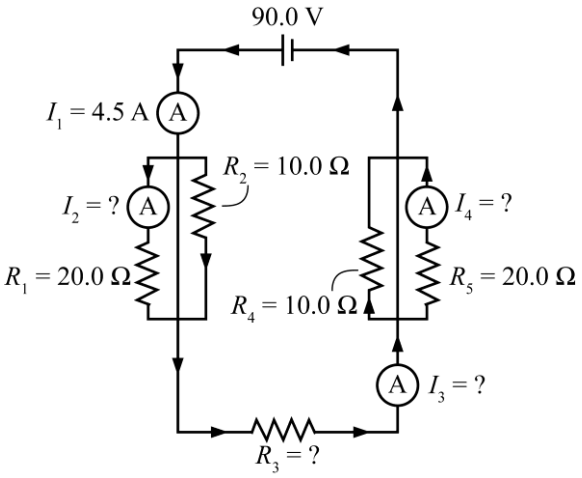
b) What is the total power dissipated by the circuit?

18. a) What is the value of I_1 in this circuit?



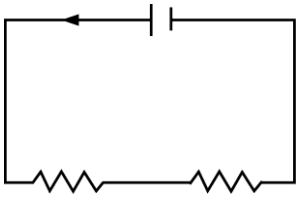
b) What is the total power dissipated in the circuit?

19. In the circuit, find R_3, I_2, I_3, and I_4.

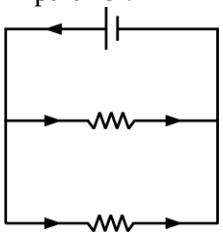


20. An electric heating coil will consume 6.0×10^2 W of power when it is connected to a 120 V outlet. A greenhouse operator has two such coils, and she wants to construct a single heater using the two coils to keep her small greenhouse at a temperature of 50.0°C during the winter months. For how many hours must this heater (two coils) operate per day if an average of 1.5×10^7 J of energy are required each winter day to maintain the desired temperature given that the coils are connected

a) in series?



b) in parallel?



c) Which of the two arrangements is the most cost-efficient? Explain your answer.

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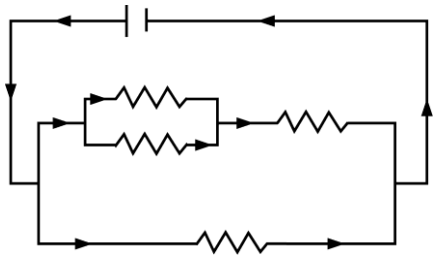
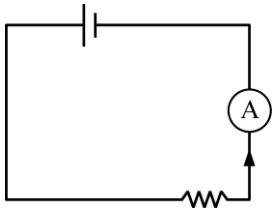
21. A creative physics student has four 12 Ω heating coils. She constructs a water heater by placing the four coils in a circuit, as shown below.



If this heater operates from a 120 V power line and is used to heat 200 kg of water (specific heat capacity = 4.18×10^3 J/kg · C) that is at an original temperature of 15° C, what will the temperature of the water be after 4.0 h? (In addition to the formulas at the back of this book, you will have to use $\Delta E = mc\Delta T$ to solve this problem. ΔE = energy used, m = mass, c = specific heat capacity, ΔT = temperature change.)

22. A physics student is testing the resistance of five equal-length conductors of 50.0 cm all made of the same material but having different cross sectional areas. She tests these conductors by constructing an electric circuit as illustrated.

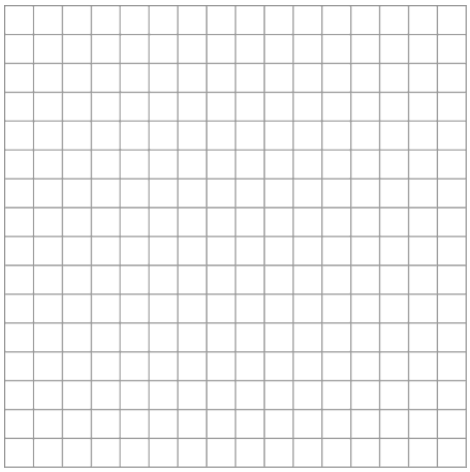


She maintains a constant voltage of 120 V across the resistors and records the current through the circuit. The following data were obtained:

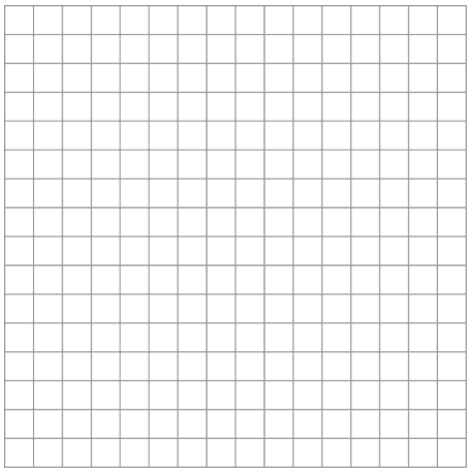
A **(× 10^{-9} m^2)**	***V*** **(V)**	***I*** **(A)**	***R*** **(Ω)**
2.1	120	30.0	
1.4	120	20.0	
1.1	120	15.0	
0.70	120	10.0	
0.35	120	5.0	

a) Complete the resistance column in the above table. (Show at least one calculation.)

b) Using your data from the complete table, plot a graph showing the relationship between the resistance and the cross-sectional area. (Plot resistance as a function of the cross-sectional area.)



c) Plot a graph of resistance against some function of cross-sectional area that will produce a straight-line graph. Label the function of the cross-sectional area you used and show your calculated value in the above data table.



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Lesson 9 ELECTROMOTIVE FORCE (EMF)

NOTES

We earlier said that a battery is a source of potential difference (voltage) or electric energy. An electric generator is another source. When these devices are not connected to an external circuit, the potential difference across the terminals is called the electromotive force or emf. This name has a historical basis. At one time, it was thought that there was something that forced the current through the circuit. Electromotive force has the symbol $\mathcal{E}$. A car battery has an emf of 12 V. When a battery is part of an electric circuit, the potential difference across the terminals of the battery is less than the emf because of the internal resistance of the battery. Every battery has some internal resistance. Because of this internal resistance, the terminal voltage is always less than the emf of the battery.

When there is no current flowing, the potential difference equals the emf.

$V = \mathcal{E} - Ir$

where V = terminal voltage

$\mathcal{E}$ = emf of battery

I = current through the battery

r = internal resistance

Note: Ir = drop in voltage due to internal resistance

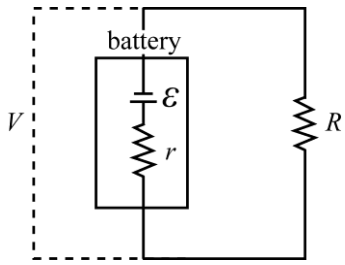
When current flows from a battery (battery discharging),

$V = \mathcal{E} - Ir$

When current flows to a battery (battery charging),

$V = \mathcal{E} \pm Ir$

Note: If the battery is not connected, no current flows and $V = \mathcal{E}$. This equation applies to an electric generator as well as a battery.



This diagram shows an external circuit with resistance R, connected to a battery. Voltage (V) across the battery terminals is given by: $V = \mathcal{E} - Ir$

When a battery goes "dead," the internal resistance becomes greater until Ir equals $\mathcal{E}$, and then the current will not flow.

When a rechargeable battery is being charged, an external voltage is applied to the battery. The voltage of this external voltage must be greater than the emf of the battery.

$V = \mathcal{E} + Ir$

V = external voltage

Example

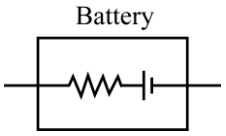
If a 12.0 V battery has an internal resistance of 0.220 Ω, what is the terminal voltage of the battery when a current of 3.00 A flows through the battery?

Solution

$$V = \mathcal{E} - Ir$$
$$= 12.0\ \text{V} - (3.00\text{A})(0.220\ \Omega)$$
$$= 11.3\ \text{V}$$

NOTES

Example



In the battery shown above, the emf is 9.0 V and when a current of 2.0 A flows through the battery, the terminal voltage is 7.0 V. What is the internal resistance of the battery?

Solution

$$V = \mathcal{E} - Ir$$

$$r = \frac{\mathcal{E} - V}{I}$$

$$= \frac{9.0\text{ V} - 7.0\text{ V}}{2.0\text{ A}}$$

$$= 1.0\ \Omega$$

Example

A 12.0 V car battery is being charged using a battery charger that is supplying 15.0 V. If the internal resistance of the battery is 1.3 Ω, what is the current through the battery?

Solution

$$V = \mathcal{E} + Ir$$

$$I = \frac{V - \mathcal{E}}{r}$$

$$= \frac{15.0\text{ V} - 12.0\text{ V}}{1.3\ \Omega}$$

$$= 2.3\text{ A}$$

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PRACTICE EXERCISES

Formula: $V = \mathcal{E} \pm Ir$

1. A flashlight battery of emf 1.5 V has an internal resistance of 0.50 Ω. If there is a current of 1.0 A through the battery, what is the terminal voltage of the battery?

2. What is the emf of a battery that has a terminal voltage of 5.0 V when a current of 1.2 A flows through the battery? The battery has an internal resistance of 0.72 Ω.

3. A battery that has an emf of 24 V and an internal resistance of 0.25 Ω is being charged at a rate of 24 A. What is the voltage required to do this?

4. A battery of 12 V emf has an internal resistance of 1.0 Ω, and is connected to an external circuit that has a resistance of 4.0 Ω. What is the current through the circuit?

5. What is the internal resistance of an electric generator that has an emf of 120 V and a terminal voltage of 115 V when there is a current of 12 A through the generator?

Lesson 10 THERMAL ENERGY

NOTES

When work is done on an object to accelerate it, the kinetic energy of the object increases.

When work is done on an object to change its vertical position, the gravitational potential energy of the object changes.

In the same way, when work is done on an object to overcome friction, the thermal energy of the object increases. When the thermal energy of an object increases, the temperature of the object increases.

Temperature is a measure of how hot or cold an object and is measured using a thermometer. The temperature of an object is an indication of the amount of thermal energy in the object.

Thermal energy is the energy that is associated with the random motion of molecules and atoms.

It has been shown that when the thermal energy of an object increases, the molecular and atomic motion in the object increases. Thermal energy is simply a measure of kinetic energy at the molecular or atomic level.

Thermal energy of an object can increase by:

- changing mechanical energy to thermal energy
- transfer of heat from one object to another

Heating an object is transferring thermal energy to the object.

HEAT

Heat is the energy that is transferred from a warm object to a cooler one. The symbol used for the change in heat energy is DE_h.

Because heat and thermal energy are types of energy, the units of heat and thermal energy are the joule (J). But you may be familiar with using *calories* and *BTUs* (British Thermal Units) as units of heat. The calorie is the heat required to raise the temperature of 1.0 g of water 1.0°C. The food calorie is equivalent to 1 000 calories. A BTU is the heat required to raise the temperature of 1.0 pounds of water 1.0°F.
1.0 cal = 4.184 J

SPECIFIC HEAT CAPACITY

Specific heat capacity is defined as the amount of heat that a unit mass of substance can gain or lose in order to change its temperature by one degree. The symbol for specific heat capacity is *c* and it is measure in joules per kilogram Celsius or:

$$\frac{\text{joules}}{\text{kilogram} \cdot \text{degree Celsius}} = \text{J}/\left(\text{kg} \cdot °\text{C}\right)$$

To increase the temperature of a substance, you increase the thermal energy of the substance by allowing heat to flow from a warmer substance.

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NOTES

Different substances have different capacities to hold heat, and therefore different specific heat capacities.

Examples of specific heat capacities:

- water has a specific heat capacity of 4 184 J/(kg·°C)
- copper has a specific heat capacity of 390 J/(kg·°C)

The heat gained or lost by a substance depends on:

- mass of substance (m)
- temperature change of the substance (ΔT)
- specific heat capacity of the substance (c)

Therefore, the change in thermal energy is defined as:

$\Delta E_{\text{h}} = m\Delta Tc$

Example

An insulated container (negligible specific heat capacity) contains 525 g of water at a temperature of 15.0°C. How much heat is needed to raise the temperature of this water to 75.0°C?

Solution

$$\begin{aligned}\Delta E_{\text{h}} &= m\Delta Tc \\ &= (0.525\ \text{kg})(75.0°\text{C} - 15.0°\text{C})\left(4.18\times10^{3}\ \text{J}/(\text{kg}\cdot°\text{C})\right) \\ &= 1.32\times10^{5}\ \text{J}\end{aligned}$$

Example

An insulated container (negligible specific heat capacity) contains 475 g of water at a temperature of 15°C. If 325 g of hot water at a temperature of 80.0°C is added, what is the temperature of the water in the container when it is completely mixed?

Solution

heat gained by cold water + heat lost by hot water = 0

or

$$\begin{aligned}\Delta E_{\text{hC}} + \Delta E_{\text{hH}} &= 0 \\ m_{\text{C}}\Delta T_{\text{C}}c_{\text{C}} + m_{\text{H}}\Delta T_{\text{H}}c_{\text{H}} &= 0 \\ m_{\text{C}}\Delta T_{\text{C}}c_{\text{C}} &= -m_{\text{H}}\Delta T_{\text{H}}c_{\text{H}} \\ (0.475\ \text{kg})(T_{\text{f}} - 15.0°\text{C})\left(4.18\times10^{3}\ \text{J}/(\text{kg}\cdot°\text{C})\right) &= -(0.325\ \text{kg})(T_{\text{f}} - 80.0°\text{C})\left(4.18\times10^{3}\ \text{J}/(\text{kg}\cdot°\text{C})\right) \\ (1986\ \text{J}/°\text{C})(T_{\text{f}} - 15.0°\text{C}) &= -(1359\ \text{J}/°\text{C})(T_{\text{f}} - 80.0°\text{C}) \\ (1986\ \text{J}/°\text{C})T_{\text{f}} - 2.979\times10^{4}\ \text{J} &= -(1359\ \text{J}/°\text{C})T_{\text{f}} + 1.087\times10^{5}\ \text{J} \\ (1986\ \text{J}/°\text{C})T_{\text{f}} + (1359\ \text{J}/°\text{C})T_{\text{f}} &= 1.087\times10^{5}\ \text{J} + 2.979\times10^{4}\ \text{J} \\ (3345\ \text{J}/°\text{C})T_{\text{f}} &= 1.385\times10^{5}\ \text{J} \\ T_{\text{f}} &= 41.4°\text{C}\end{aligned}$$

NOTES

Example

When 0.300 kg of lead ($c = 1.30 \times 10^2$ J/(kg·°C)) at a temperature of 165°C is added to water at a temperature of 15.0°C, the temperature of the water raises to 25.0°C. What is the mass of the water? (Assume the water is in a container that has negligible heat capacity.)

Solution

heat gained by water + heat lost by lead = 0

or

$$\Delta E_{\text{hW}} + \Delta E_{\text{hL}} = 0$$

$$\Delta E_{\text{hW}} = -\Delta E_{\text{hL}}$$

$$m_{\text{W}} \Delta T_{\text{W}} c_{\text{W}} = -m_{\text{L}} \Delta T_{\text{L}} c_{\text{L}}$$

$$m_{\text{W}} (25.0°\text{C} - 15.0°\text{C})\left(4.18 \times 10^3 \text{ J}/(\text{kg} \cdot °\text{C})\right) = -(0.300 \text{ kg})(25.0°\text{C} - 165°\text{C})\left(1.30 \times 10^2 \text{ J}/(\text{kg} \cdot °\text{C})\right)$$

$$m_{\text{W}} (10.0°\text{C})\left(4.18 \times 10^3 \text{ J}/(\text{kg} \cdot °\text{C})\right) = -(0.300 \text{ kg})(-140°\text{C})\left(1.30 \times 10^2 \text{ J}/(\text{kg} \cdot °\text{C})\right)$$

$$\left(4.180 \times 10^4 \text{ J/kg}\right) m_{\text{W}} = -5.460 \times 10^3 \text{ J}$$

$$m_{\text{W}} = \frac{-5.460 \times 10^3 \text{ J}}{4.180 \times 10^4 \text{ J/kg}}$$

$$m_{\text{W}} = 0.131 \text{ kg}$$

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PRACTICE EXERCISES

Formula: $\Delta E_h = m\Delta Tc$

1. An insulated container (negligible specific heat capacity) contains 462 g of water at a temperature of 24.0°C. How much heat is needed to rise the temperature of the 462 g of water from 24.0°C to 80.0°C?

2. How much heat is needed to raise the temperature of 462 g of lead ($c = 1.30 \times 10^2$ J/(kg·°C)) from 24.0°C to 80.0°C?

3. What is the final temperature of 0.200 kg of a clear liquid ($c = 3.47 \times 10^3$ J/(kg·°C)) at an original temperature of 20.0°C when it gains 2.50×10^4 J of heat?

4. What mass of water can be heated from 25.0°C to 40.0°C using 2.10×10^4 J of heat?

5. An insulated container (negligible specific heat capacity) contains 185 g of water at a temperature of 12.0°C. If 295 g of water at a temperature of 85.0°C is added, what is the final temperature of the mixture when it is completely mixed?

6. A 0.240 kg copper mass ($c = 3.90 \times 10^2$ J/(kg·°C))is heated to 215°C and quickly placed in an insulated container (negligible specific heat capacity) that contains 0.275 kg of water that has a temperature of 12.0°C. What is the final temperature of the water?

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7. A student did an experiment to determine the specific heat capacity of an unknown metal. She heated 0.352 kg of the metal to 215°C and quickly placed it in an insulated container (negligible specific heat capacity) that contained 0.265 kg of water at a temperature of 26.0°C. If the final temperature of the water is 33.0°C, what is the specific heat capacity of the metal?

8. A rain drop (mass = 1.00 g) is travelling at a speed of 40 m/s when it hits the surface of 100 g of water contained in a drinking glass. Assume that all the kinetic energy of the raindrop is converted to energy that changes the water's temperature and the water in the glass and the raindrop were at the same original temperature. What is the change in temperature of the water in the glass?

9. 20.0 g of an unknown liquid at a temperature of 10.0°C is mixed completely with 29.0 g of the same liquid at a temperature of 52.0°C. What is the temperature of the mixture?

PRACTICE TEST

1. Describe two possible consequences to energy when work is being done on an object.

2. A physics student raises a box at a constant velocity and places it on a frictionless table. He then accelerates the box to 4.0 m/s. How do the object's potential and kinetic energy change during this event?

3. A person exerts a force on a heavy object to hold it 1.5 m above the ground. Why is no work done on the object?

4. Give an example of a non-conservative force and describe why it is non-conservative.

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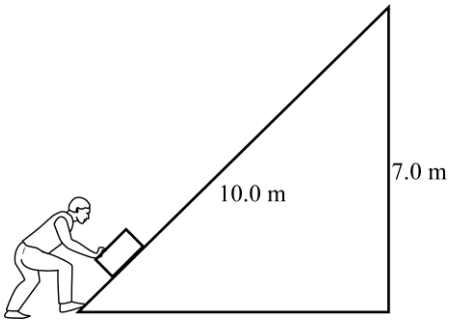
5. A heavy box is pushed up an incline at a constant velocity as shown in the diagram.



Sketch a graph that represents the force-displacement graph for this motion assuming that the friction is constant along the incline.

6. Noah dragged a heavy object over a horizontal surface at a constant velocity overcoming the force of friction. After a while, he notices that the object is hot. Why does the object Noah drags become hot?

7. When the velocity of an object is halved, what happens to its kinetic energy?

8. When the force exerted on an object is plotted on a graph versus the object's displacement, what property on the graph represents the work done on the object?

9. Give an example of a conservative force and describe why it is conservative.

10. Elissa runs up a flight of stairs with a velocity of v. Her gravitational potential energy at the top of the stairs is E_p.

a) What is an expression that represents the student's gravitational potential energy at the top of the stairs if she had run up at a velocity of $2v$ instead?

b) If Elissa exerted an average power of P to reach the top of the stairs, what is an expression that represents the power that she exerted if she had run up the stairs at a velocity of $2v$ instead?

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11. How much power is required to lift a 45 kg student vertically at a speed of 0.25 m/s?

12. A cup of hot water can be cooled down by putting a strip of cold metal into it. Suppose you have a strip of each of the following metals. Each strip is of equal mass and temperature.

Metal	Specific Heat Capacity
aluminum	880 J/(kg·°C)
copper	390 J/(kg·°C)
silver	230 J/(kg·°C)
lead	130 J/(kg·°C)

Which of these metals would cool the water the most?

13. A physics student has two bars of unknown metal both at room temperature. Each bar has a mass of 1.0 kg. She applies 1.0×10^4 J of heat to each bar of metal. Metal bar 1 became warm to the touch while metal bar 2 became very hot. Why are the metals at a different temperature?

14. Elias mixes 1.0 kg of liquid X (temperature = 86°C) with 2.0 kg of liquid Y (temperature = 24°C). He measures the final temperature of the mixture at 62°C. Which of the two liquids has the higher specific heat capacity?

15. Shareen mixes 1.0 kg of liquid X (temperature = 10°C) with 2.0 kg of liquid Y (temperature = 58°C). She measures the final temperature of the mixture at 34°C. Which of the two liquids has the higher specific heat capacity?

16. An object is thrown vertically into the air. Sketch a graph that represents the relationship between the kinetic energy and the gravitational potential energy of the object as it reaches its maximum height assuming negligible air resistance.

17. An object is dropped from a certain height above the ground. Sketch a graph that represents the relationship between the kinetic energy and the velocity of the object assuming negligible air resistance.

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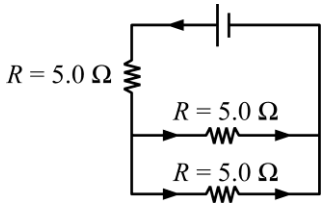
18. Why is a machine never 100% efficient?

19. What is the potential difference across a 5.0 Ω resistor if it carries a current of 9.0 A?

20. What is the total number of electrons that passes through a conductor each minute if it carries a current of 0.20 A?

21. If a 775 W heating coil operates for 15 minutes, how much energy does it use?

22. An electric circuit has three resistors as shown in the following diagram.



What is the equivalent resistance of the three resistors?

23. Two resistors having values of 5.00 Ω and 7.50 Ω are connected in series in an electric circuit. What is their equivalent resistance?

24. What instrument would you use to determine the resistance through an electric circuit?

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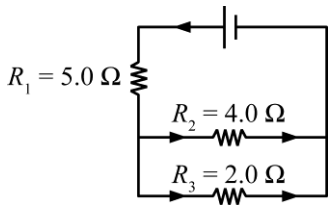
25. An electric circuit has three resistors, as shown in the diagram below.



If the current through R_1 is 9.0 A, what is the current through R_2?

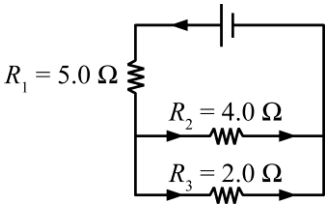
26. What property will two resistors connected in parallel share in an electric circuit?

27. What property will two copper conductors share if they have the same resistance?

28. An electric current flows through a resistor R_1 that is in an electric circuit. If a second resistor R_2 is now placed in the circuit in series with R_1, what happens to the current through R_1?

29. A resistor R_1, which is placed in an electric circuit, dissipates energy at a rate of P. If a second resistor R_2 is now placed in the circuit in parallel with R_1, what happens to the energy dissipated in R_1?

30. An electric circuit has three resistors as shown in the diagram below.



a) What is the total resistance of this circuit?

b) What is the terminal voltage of the battery if there is a current of 1.6 A flowing through the circuit?

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WAVES AND SOUND

BIG IDEA
Mechanical waves transfer energy but not matter..

As you begin this unit, you should consider the following questions…

- What are the factors that affect wave behaviours?
- How would you investigate the relationships between the properties of a wave and properties of the medium?
- How can you determine which harmonics are audible in different musical instruments?
- What data are needed to determine the speed of sound in air?

Lesson	Page	Completed on
1. Wave Characteristics	224	
2. Properties of Waves	232	
3. Sound	240	
4. Characteristics of Sound	258	
5. Air Columns	271	
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- When you are finished this unit, you are expected to know …
- The **generation and propagation of waves,** and indentifying **transverse versus longitudinal** waves and **linear versus circular** waves
- Recognize both the **properties (differences between the properties of a wave and the properties of the medium, periodic versus pulse)** and **behaviours (reflection (open and fixed end), refraction, transmission, diffraction, interference, Doppler shift, standing waves, interference patterns, law of superposition) of waves**
- Identify the **characteristics (for example, pitch, volume, speed, Doppler effect, sonic boom) of sound**
- The **resonance and frequency (for example, harmonic, fundamental/natural, beat frequency) of sound**

Lesson 1 WAVE CHARACTERISTICS

NOTES

ENERGY: ITS TRANSMISSION BY WAVES

Energy can be transferred by means of either waves or particles. If you get hit with a ball at a baseball game, the energy transfers from the bat to you by means of a particle, the ball. When you hear the crack of the bat against the ball, the energy from the bat travelled to you as a sound wave.

Energy can be transmitted as mechanical waves. Sound and water waves are examples of energy transmitted by waves, and these examples can be used to demonstrate the properties of waves.

DEFINITION OF A WAVE

A wave is a disturbance (vibration) that carries energy from one point to another without the transmission of matter (particles). The disturbance is transmitted; the matter through which the disturbance propagates is not transmitted.

Energy is required to form a wave or disturbance in a medium, so a wave or vibration travelling through a medium is equivalent to the transmission of energy through the medium.

Any wave that requires a material substance for its propagation (transmission) is defined as a mechanical wave. Sound waves, water waves, and earthquake tremors are examples of mechanical waves. Not all waves are mechanical waves.

TYPES OF WAVES

Types of waves:
–Mechanical
–Electromagnetic

Until the 1860s, it was believed that all waves were mechanical waves. A mechanical wave is the vibration of a medium (air, water, string, etc.) in response to a disturbance of the medium at some point. The vibration of the medium transmits the energy of the disturbance.

Types of mechanical waves:
–Longitudinal
–Transverse

During the 1860s, James Clerk Maxwell proposed that waves could travel by vibrating electric and magnetic fields. These waves are called electromagnetic waves. Mechanical and electromagnetic waves have common properties, but there is one important difference. While mechanical waves require a medium, electromagnetic waves do not; they can travel in a vacuum.

The vibration of the medium in mechanical waves is in simple harmonic motion. The direction of the vibration can be parallel or perpendicular to the direction of energy flow. The direction of the vibration determines whether the mechanical wave is longitudinal or transverse.

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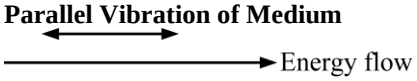
NOTES

LONGITUDINAL WAVES

If the vibration is parallel to the direction of energy flow, the wave is **longitudinal**. Sound is an example of energy that travels in longitudinal waves.

Parallel Vibration of Medium

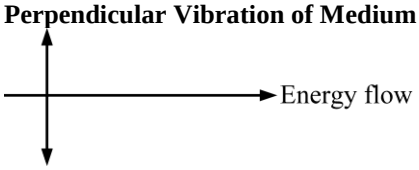


In a longitudinal wave, the points at which the medium is densest are called **compressions**. The points at which the medium is least dense are called **rarefactions**. A longitudinal wave travels as a periodic oscillation of compressions and rarefactions. This motion is often described as back and forth motion.

TRANSVERSE WAVES

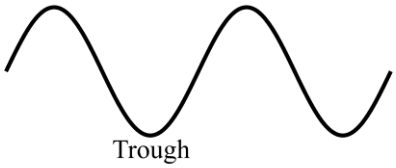
If the vibration is perpendicular to the direction of energy flow, the wave is **transverse**. Water waves are largely transverse, but they have a small longitudinal component.

Perpendicular Vibration of Medium



The maximum displacement of a medium from its rest position in a transverse wave is called a **crest** or **trough**. A trough is the maximum displacement of the medium in the opposite direction of the crest.

Transverse wave



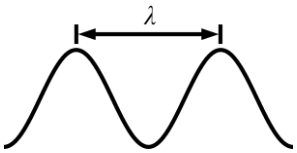
Longitudinal wave

Every point on the wave vibrates up and down (or back and forth) like a swinging pendulum.

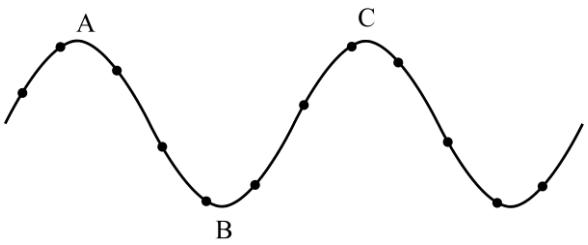
NOTES

BASIC CHARACTERISTICS OF WAVES

All waves share basic characteristics. All waves have a characteristic **wavelength**, which is the distance between two adjacent points in phase on a wave. It is the distance from one crest to the next crest or the distance from one compression to the next. The Greek letter lambda, λ, is the symbol for wavelength.



Waves are described as being **in phase** when two points in a medium are vibrating together. Soldiers march in phase. They move their left feet together, and then they move their right feet together. In the given diagram, the points A and C of the wave are in phase.



$f = \frac{1}{T}$

When two points in a medium are not vibrating together, they are **out of phase**. Students walking the halls at school are walking out of phase because when one student moves her left foot, another student might move his right foot; they are not moving together at the same time. In the given diagram, B and C in the wave are out of phase.

The time it takes one point in a wave to travel one wavelength, or the time of one full vibration of a medium, is called the **period**. The symbol for period is *T*, and the unit of measure is usually seconds (s). The number of waves that pass a point in a given time, or the number of vibrations of a point in the medium in a given time, is known as the **frequency**. Frequency is represented by the symbol *f*, and it is measured in units of reciprocal seconds (1/s) or hertz (Hz). A frequency of 2 Hz means that some event occurs twice in one second (2 vibrations/s, 2 rotations/s, 2 revolutions/s, etc.). There is an inverse relationship between the period and frequency of a wave.

$T = \frac{1}{f}$ $\quad$ $f = \frac{1}{T}$

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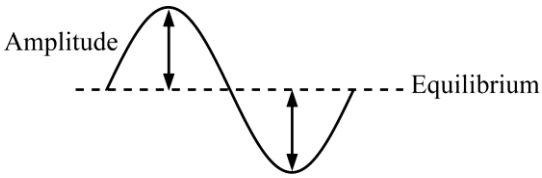
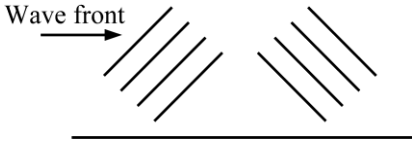
NOTES

The maximum displacement of the medium from its equilibrium position is the **amplitude** of the wave. The amplitude can be measured at any crest or trough.

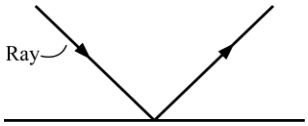


A **pulse** is a wave of short duration or a single vibration that travels through a medium. If you slap your desk, you create a single compression or pulse.

The **wave front** is used to study waves on a surface. Diagrams representing simple waves generated on a surface will show the wave front as a straight line or the arc of a circle. This diagram is called a wave front diagram.



A **ray diagram** is a diagram used to show the direction that energy is carried by a wave as it travels from a source to a receiver.



The direction of motion of the wave (the ray) is always perpendicular to the wave front, whether the wave is straight or circular.

SPEED OF A WAVE THROUGH A MEDIUM

Just as the speed of a horse in a horse race depends on the nature of the track (the conditions), the speed of a wave depends on the nature of the medium through which it is travelling.

NOTES

In general terms, the speed of a wave through a medium depends on the elasticity and the density of the medium. **Elasticity** is the ability of a substance to maintain its shape when subjected to a deforming force and its ability to recover its original shape after the stress is removed. For example, steel has a high elasticity. Phase affects elasticity in a significant way. Particles (atoms and molecules) in solid phase are in close contact and interact strongly with each other. This is less true for liquids. In the case of gases, the individual particles act independently of one another. The magnitude of these particle interactions affect elasticity. Consequently, solids are more elastic than liquids, which are more elastic than gases. The speed of a wave will be higher when the elasticity is higher.

The speed of a wave depends on the nature of the medium through which it is travelling.

Wave speed also depends on the inertia of the component atoms or molecules. The inertia of these atoms and molecules is a measure of their responsiveness (or lack of responsiveness) to any attempt to change their position. Inertia is a property of mass, and the more massive the atoms and molecules, the more sluggish their response to a wave disturbance will be. This inertial property of matter is described by **density**. For a given phase, density becomes an important factor in influencing wave speed: the wave speed in the medium will decrease as the density of the medium increases.

Speed of sound (at 25°C)
in steel: 5960 m/s
in water: 1498 m/s
in dry air: 346 m/s

$$v = \sqrt{\frac{\text{elasticity}}{\text{density}}}$$

Factors that affect the elasticity or density of the medium, such as temperature, will also affect the speed of the wave being propagated. For example, the speed of sound through 0°C dry air is approximately 331 m/s, while the speed of sound through 25°C dry air is about 346 m/s.

The two different measurements for the speed of sound in air can be explained by considering which property is affected by the temperature of the medium, and then considering what can be inferred about the wave speed using the formula above. As the temperature of the air increases, its density decreases because temperature is a measure of the average kinetic energy of atoms and molecules in the air. As the particles move faster, they move apart and become less dense. The speed of the wave is indirectly related to the density, so as the density decreases, the speed of the wave should increase.

Universal wave equation
$v = \lambda f$

The speed of a wave can also be determined from its wave characteristics (wavelength and frequency). The speed, wavelength, and frequency of a wave are related according to the universal wave equation.

$$v = \lambda f$$

The universal wave equation is a special form of $v = \frac{d}{t}$.

This equation is really a form of the definition of uniform motion.

$$v = \frac{d}{t}$$

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NOTES

The distance from one crest to the next crest is one wavelength. The time it takes a wave to travel the distance equal to one wavelength is the period, *T*, of the wave.

$$v = \frac{\lambda}{T},$$

but $T = \frac{1}{f}$;

therefore, $v = \lambda f$

Example

A wave has a frequency of 2.10 Hz and a wavelength of 5.30 m. What is its speed?

Solution

$$\begin{aligned} v &= \lambda f \\ &= (5.30 \text{ m})(2.10 \text{ Hz}) \\ &= 11.1 \text{ m/s} \end{aligned}$$

Example

A wave has a speed of 5.00×10^{-1} m/s and a wavelength of 2.00 m. What is its period?

Solution

$$v = \lambda f \Rightarrow f = \frac{v}{\lambda}$$

$$\begin{aligned} T &= \frac{1}{f} \\ &= \frac{\lambda}{v} \\ &= \frac{2.00 \text{ m}}{5.00 \times 10^{-1} \text{ m/s}} \\ &= 4.00 \text{ s} \end{aligned}$$

PRACTICE EXERCISES

Formulas: $T = \frac{1}{f}$ $v = \lambda f$ $v = \frac{d}{t}$

Use the following information to answer the next question.

A wave has a frequency of 5.0×10^{-1} Hz and a speed of 3.3×10^{-1} m/s.

1. What is its wavelength?

2. What is the speed of a wave if it travels 4.60 m in 2.00 s?

Use the following information to answer the next question.

A water wave has a wavelength of 5.0 m and a speed of 2.5 m/s.

3. What is its period?

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4. If 9.5 waves break on the beach in 1.00 min, what is the frequency of these waves?

5. If sound waves travel at 335 m/s, what wavelength of sound has a period of 1.00×10^{-2} s?

Use the following information to answer the next question.

A radio station broadcasts at a frequency of 1.00×10^{6} Hz.

6. If the speed of this wave is 3.00×10^{8} m/s, what is its wavelength?

Use the following information to answer the next question.

A light wave has a speed of 3.00×10^{8} m/s.

7. If the length of this wave is 5.00×10^{-7} m, what is its period?

Lesson 2 PROPERTIES OF WAVES

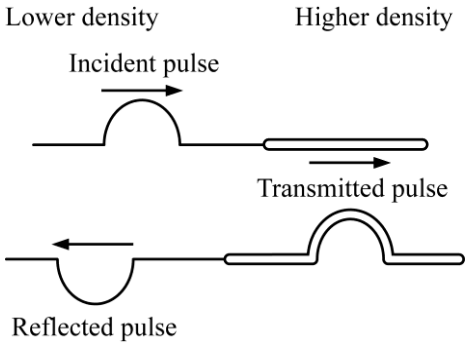
NOTES

WAVES AT A BOUNDARY

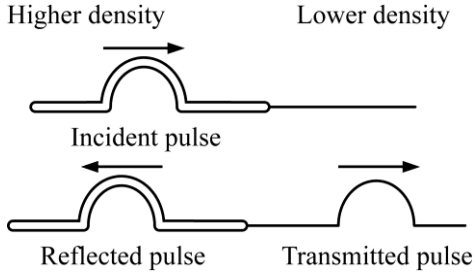
Both waves and particles will reflect and refract, but only waves will diffract and undergo constructive and destructive interference. Diffraction and interference are fundamental properties of waves.

Notice that when waves travel from one medium to another, some of the energy is transmitted and some is reflected. The transmitted pulse is always erect, but the orientation of the reflected pulse depends on the media. When the pulse travels from a medium with a lower density to a medium with a higher density, the reflected pulse is inverted with respect to the incident pulse. However, when the pulse travels from a medium with a higher density to a medium with a lower density, the reflected pulse is erect (unchanged in orientation with respect to the incident pulse).

1. **a)**



2. **a)**



In all cases, the transmitted pulse has the same orientation as the incident pulse.

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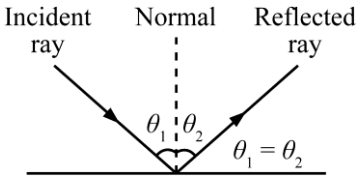
REFLECTION OF WAVES

When a wave reaches a boundary between two media, some or all of the wave reflects. Particles also reflect. An example that demonstrates this property is a ball bouncing on the floor. The law of reflection states that the angle of reflection (θ_r) equals the angle of incidence (θ_i). The given ray diagram shows the law of reflection.

NOTES

Law of reflection:
$\theta_i = \theta_r$

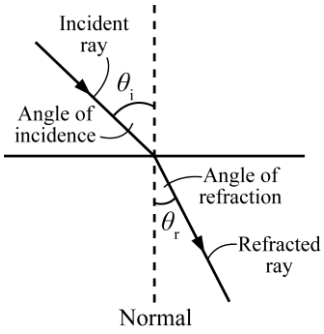


Incidence means incoming. The normal is a line perpendicular to the reflecting surface. The angle of incidence and the angle of reflection are measured with respect to the normal.

Knowledge of sound properties is useful for acoustics and other technologies. In the construction of a concert hall, the builder wants to eliminate echoes. Echoes are simply the reflection of sound. Sonar devices are used to detect the presence of objects (fish, submarines, etc.) in water. These devices emit a sound wave and detect the reflected wave. By knowing the time between the transmission and reflection, it is possible to determine the depth of the object in the water. Mirrors and satellite dishes also make use of reflection.

REFRACTION OF WAVES

Refraction occurs when a wave changes the direction in which it is moving. This is caused by a change in speed as the wave passes from one medium to another.



The angle of incidence and the angle of refraction are both measured from the normal.

Similar to reflection, the angles of incidence and refraction are measured with respect to the normal.

NOTES

Although the relationship between these two angles is not easy to see, Willebrord Snellius, a Dutch mathematician, was able to determine it. This relationship is referred to as the law of refraction or Snell's law.

$$\frac{\sin\theta_i}{\sin\theta_r} = n(\text{constant}) = \frac{n_r}{n_i}$$

Law of refraction (Snell's law):
$$\frac{\sin\theta_i}{\sin\theta_r} = n$$

The constant *n* is called the index of refraction. It is equal to the ratio of the sine of the incident angle to the sine of the refracted angle. For a given pair of materials, the index of refraction is constant for a wave with a given frequency.

Refraction is caused by a change in speed.

Refraction is the result of a change in the speed of the wave as it enters a different medium. Measurements show that the ratio of the speed in the incident medium (first medium) compared to the speed in the refracting medium (second medium) is equal to the ratio of the sine of the angles.

$$\frac{v_i}{v_r} = \frac{\sin\theta_i}{\sin\theta_r}$$

$$\frac{v_i}{v_r} = \frac{\sin\theta_i}{\sin\theta_r} = n = \frac{n_r}{n_i}$$

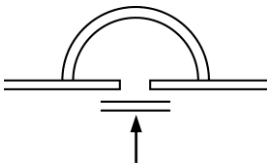
The index of refraction can be defined as the ratio of the wave speeds in the two media. A larger change in wave speed results in a larger magnitude for the index of refraction. Also, the greater the change in direction of the wave, the greater the index of refraction.

When waves pass from one medium to another, the direction, speed, and wavelength change, but the frequency does not.

DIFFRACTION OF WAVES

Waves diffract. Particles do not diffract.

Waves have the ability to undergo diffraction. Diffraction is the spreading or bending of a wave as it passes through a small opening or moves past an obstacle.



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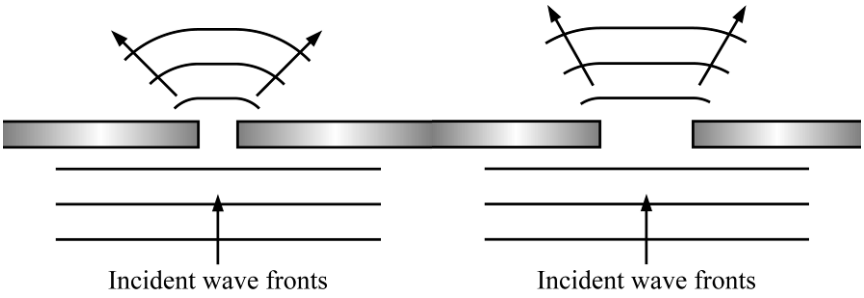
NOTES

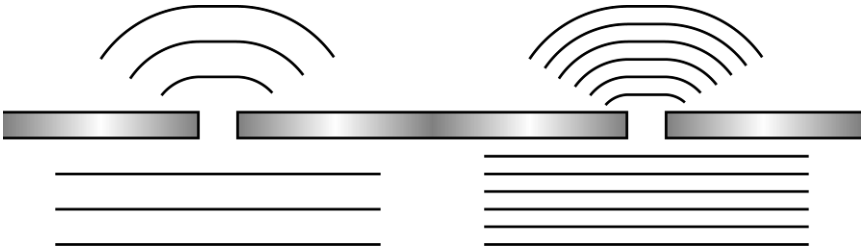
The amount of spreading out (diffraction) depends on the following factors:

- The size of the opening—the smaller the opening, the greater the diffraction.

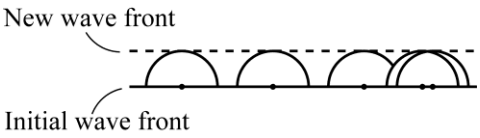


The longer the wavelength, the greater the diffraction.

- The wavelength—the longer the wavelength, the greater the diffraction.



Christiaan Huygens explained that if the source of a wave disturbance was a single point, the waves produced would be circular. Drop a pebble in a pool of still water, and the circular wave front produced is apparent. From this observation, Huygens proposed that every point on a wave front is the source of a new circular wave front. A straight wave front would propagate as shown in the given diagram.



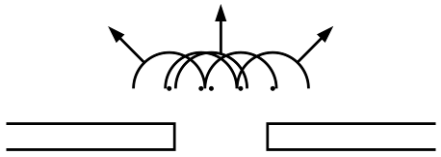
An infinite number of points along the wave front all behave as tiny point sources of an infinite number of circular wave fronts. The net effect is a new, straight wave front that is indistinguishable from the initial wave front.

The direction of motion of the wave front is perpendicular to the wave front.

This characteristic of waves, called the Huygens principle, can be extended to clarify diffraction. The part of the wave travelling near the middle of an opening continues as a straight wave because the infinite number of point sources regenerate the straight wave. However, the points at the very edge of the opening produce circular wave fronts that do not merge with those coming from the adjacent points.

NOTES

As the point sources on the ends of the wave spread apart, the wave front bends and becomes circular.

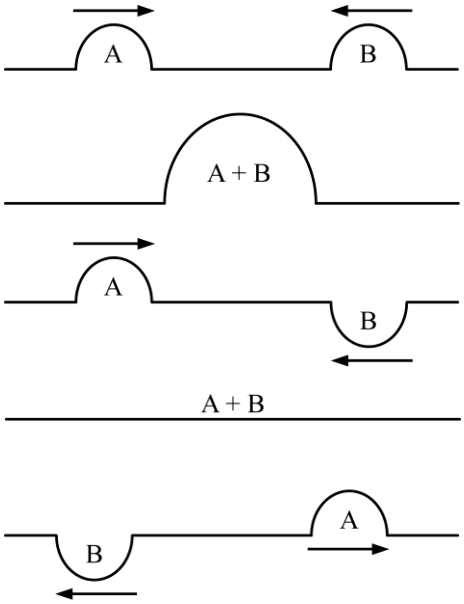


SUPERPOSITION PRINCIPLE

Both waves and particles will reflect and refract, but only waves will diffract and undergo constructive and destructive interference. Diffraction and interference are fundamental properties of waves.

The superposition principle states that when two or more waves meet, the amplitudes of these waves will combine. The resultant wave amplitude is the algebraic sum of the individual wave amplitudes.

Consider the simple example of two transverse wave pulses travelling toward each other on a spring or rope.



The superposition principle explains constructive and destructive interference.

When a crest from one source meets a crest from another source in one medium, the energies add together to displace the medium farther. Likewise, when a trough meets a trough, the amplitude of each wave is combined. This is called constructive interference. When a crest and trough meet, the energies tend to cancel. If the amplitude directed upward is viewed as positive, and the downward directed amplitude is considered negative, the algebraic addition of these two pulses results in a smaller amplitude. This is called destructive interference.

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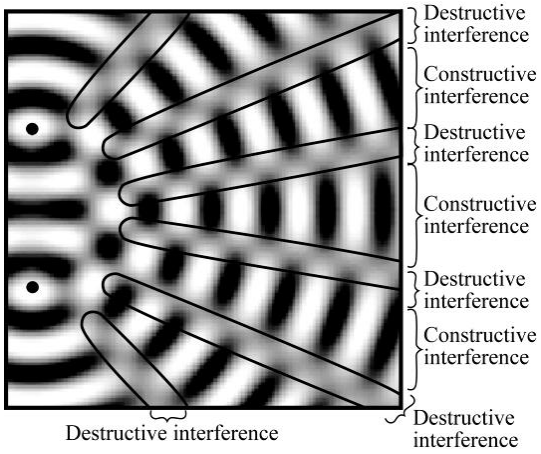
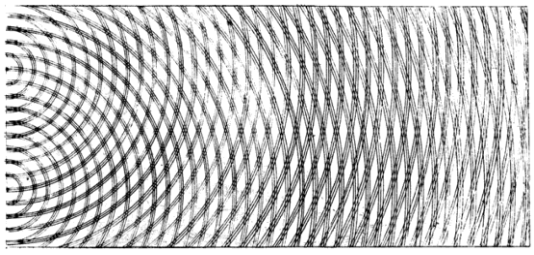
In each case, the pulses pass through each other. Each pulse influences the other as it passes through the other pulse.

Waves in water help explain how sound waves behave. Consider two point sources that are in phase, creating circular wave fronts in a ripple tank. The waves will be superimposed on one another and produce an interference pattern. The pattern will produce both constructive interference (maximums) and destructive interference (minimums).



If points of constructive interference are connected, antinodal lines are created. If points of destructive interference are connected, nodal lines are created.

The constructive and destructive interference of sound waves results in beats. If the sound waves combine in phase, there is constructive interference. If the sound waves combine out of phase, there is destructive interference. Beats can be produced by using two tuning forks of slightly different frequencies. As the waves move together both in and out of phase, the sound will reach a maximum (constructive interference) and then a minimum (destructive interference).



NOTES

Constructive interference: maximum amplitude (anti-node)

Destructive interference: minimum amplitude (node)

Nodal lines: destructive interference (a minimum)

Antinodal lines: constructive interference (a maximum)

ACTIVITY 1

Purpose

To examine the properties of transverse waves reflected at a boundary.

Apparatus

Two springs of different densities.

Procedure

1. Connect the two springs together and stretch this combination.
2. Create a pulse at one end of the combined spring.
3. Observe the pulse as it reaches the boundary between the two smaller springs.

Question

What difference exists between the reflected pulse of waves that originate from the lower density and higher density springs?

ACTIVITY 2

Purpose

To examine the superposition of transverse waves.

Apparatus

A long spring

Procedure

1. Lay the spring on a flat, horizontal surface.
2. With a partner, stretch the spring until it becomes reasonably straight (i.e., apply tension to the spring).
3. Create a transverse pulse in the spring towards your partner and have him or her do the same.
4. Observe what happens when the pulses meet.

Question

What conclusions can be drawn from the observations?

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PRACTICE EXERCISES

Calculate the amplitude of the combined wave when waves with the following amplitudes combine in phase.

1. 2.3 cm and 1.9 cm

2. 5.2 cm and 2.5 cm

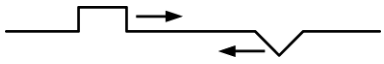
Calculate the magnitude of the amplitude of the combined wave when waves with the following amplitudes combine out of phase.

3. 2.3 cm and 1.9 cm

4. 5.2 cm and 2.5 cm

Sketch the shape of the superposed amplitude if the two given waves overlap completely.

5.



6.

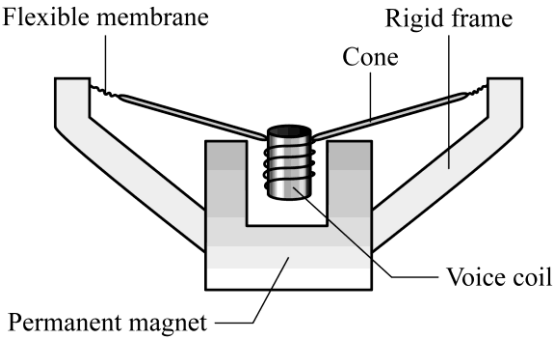
7.

Lesson 3 SOUND

NOTES

People are very reliant on sound. Sound travels through air as a longitudinal wave. In your study of sound, you will see how its wave characteristics are responsible for what you hear.

The compressions and rarefactions that form as a sound wave travels are really alternating regions of high and low pressure. Technological innovation has created devices, such as a microphone, that respond to these pressure levels and convert them into other forms of energy. A microphone consists of a thin cone or disk attached to a rigid frame by a flexible membrane. This structure can move in response to the pressure changes associated with a sound wave.



When sound waves strike the cone, they move it back and forth. A very thin wire coil is attached to the cone, and this coil moves back in forth within the uniform magnetic field produced by a strong permanent magnet. Electrical current is induced in the voice coil, and the amplitude of the current varies in perfect correspondence with the amplitude (pressure) changes in the original sound wave.

Loud speakers and headphones work like microphones, only in reverse. An electrical signal is input into the speaker, whose amplitude variations are a precise match of the original sound. The voice coil moves in response to this electrical signal, and this motion is passed on to the speaker cone. The pressure changes are heard as sound.

The ear converts the mechanical energy of sound waves into the electrochemical energy of nerve impulses. The outer ear (or pinna) is dish-shaped, and there are a number of curves and ridges that help collect the sound energy and enable humans to determine where a particular sound wave originates. Your brain compares the signal from each ear and uses differences in intensity and time to locate the position of the sound source.

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The collected sound enters the auditory canal. This canal is separated from the inner ear by the tympanic membrane (eardrum). The pressure in the inner ear is maintained at normal atmospheric pressure because it is connected to the outside by the Eustachian tube. A compression (region of increased air pressure) that meets the tympanic membrane pushes the membrane inward. A rarefaction (region of reduced air pressure) will result in the membrane moving outward because the pressure in the middle ear is now larger.

The middle ear contains three delicate bony structures that link the tympanic membrane to the oval window. The oval window separates the middle ear from the inner ear, or cochlea. These three bones (the hammer, anvil, and stirrup) accurately amplify and transmit the vibrations of the tympanic membrane to the oval window. This amplification is extremely effective. The pressure wave transmitted through the fluid in the cochlea is about 22 times the pressure felt at the eardrum.

The cochlea is filled with fluid and lined with between 20 000 to 30 000 reed-like fibres. Fibres at different locations in the cochlea respond to different frequencies. Different fibres have different lengths and stiffnesses. This gives them specific natural resonant frequencies and the ability to selectively respond to sounds of a particular frequency. When a reed-like fibre is put into resonance by a particular sound frequency, it stimulates tiny hair cells called cilia. The motion of the cilia results in nerve impulses, which are sent to the brain. Human understanding of the structure and function of the cochlea is far from complete, but the complexity and ingenuity of the organ is astonishing.

AUDIO ENGINEERING

Scientists, electrical and acoustical engineers, audiologists, and other specialists are driven by the objective to reproduce sound in such a way that the listener cannot distinguish the reproduction from the original source.

Although great strides in music fidelity have been made, much remains to be done. Electronic equipment capable of reproducing frequencies from 20 Hz to 20 000 Hz with distortions that are lower than our ability to hear them now exists. A good set of earphones and an MP3 player no bigger than a deck of cards combine to deliver truly astonishing audio fidelity.

A recent technological development is noise-cancelling headphones. Headphones are like miniature loudspeakers and operate on similar principles. Both are called transducers because they convert a transverse electrical signal into a longitudinal sound wave. Unfortunately, in addition to the music from the headphone, there are often ambient sounds that can reduce the fidelity of the music. In-ear headphones are designed for insertion into the ear canal and the snug fit ensures a good bass response and attenuates high frequency ambient noise. However, low frequency noise is difficult to reduce in this way.

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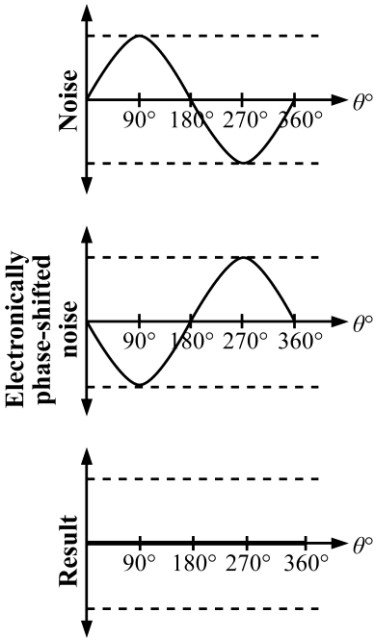
Noise-cancelling headphones are equipped with electronic circuits that detect low frequency noise and then phase-shift this noise by 180°. This altered signal is then mixed with the original music signal, so that the noise is cancelled through destructive interference.



The diagram shows the original noise alongside the electronically phase-shifted noise. The destructive interference that occurs is easier to visualize. Ambient noise and high fidelity music are much more complex waveforms. In real time, this entire process happens very rapidly, and the noise reduction is approximately 15–20 dB.

SPEED OF SOUND

Recall that the elasticity of gases is very low. However, this factor does not affect the wave speed of sound in gases in a significant way. The density of a gas, on the other hand, is far more important.

The speed of sound is faster in liquids than in gases, and it is faster still in solids.

The speed of sound in air depends on the temperature of the air because the speed of a wave is related to the density of the medium. In this case, the medium is air. As the temperature of the air increases, its density decreases.

$$v = \sqrt{\frac{\text{elasticity}}{\text{density}}}$$

From the given formula, you can see that as the density decreases, the wave speed increases.

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The speed of sound in dry air at room temperature (20°C) is approximately 343 m/s. The speed of sound through air at 0°C is about 331 m/s. For each °C rise in the temperature of the air, the speed of sound increases by 0.59 m/s. The speed of sound in dry air at temperatures reasonably close to 0°C can be found using the following formula:

$$v = \left(331 + \left(0.59 / °C\right)T\right) \text{ m/s},$$

where $T =$ temperature in °C

NOTES

The speed of sound in dry air at room temperature is approximately 343 m/s.

Ultrasound—frequency greater than 20 000 Hz

SOUND AT A BOUNDARY

The human ear is sensitive to frequencies between 20 Hz and 20 000 Hz. However, this upper limit drops as humans age. Ultrasound is sound that has a frequency greater than 20 000 Hz. It should be pointed out that animals such as dogs, bats, and dolphins can hear sound with frequencies well beyond 20 000 Hz. Sonar devices usually use ultrasonic frequencies.

ECHOLOCATION OF BATS

Bats are active from dusk to dawn. In little or no light, they are able to locate tiny flying insects, their predominant source of food. They do this using echolocation, which is the location of objects by reflected sound.

If you stood in front of a cliff and gave a loud shout of short duration, an echo would be heard a short time later.

How a bat can process echo information and retrieve this kind of distance information is not well understood. Bats make ultrasonic sounds by forcing air past their larynx. This sound reflects off something, such as an insect, and returns to the bat. Without a stopwatch and calculator, the bat is able to determine the location of the insect. This process is repeated on the fly, with the bat moving in a complex flight path as it responds to an erratically moving insect.

It is believed that bats can infer information about the insect's size from the intensity of the reflected sound. A large insect would reflect more sound energy than a small one. Even more impressive, it is also believed that the bat can tell if the insect is moving toward it, away from it, or laterally with respect to the bat's location. If the reflected sound has a higher frequency, the insect is travelling toward the bat.

Example

A student stands in front of a tall building and measures the time delay between a shout and the echo to be 1.53 s. If the air surrounding the student is at 5.00°C, how far away is the building located?

NOTES

Solution

$$\begin{aligned} v &= \left(331 + \left(0.59/°\text{C}\right)T\right) \text{ m/s} \\ &= \left(331 + \left(0.59/°\text{C}\right)(5.00°\text{C})\right) \text{ m/s} \\ &= 334.0 \text{ m/s} \\ d &= vt \\ &= \left(334.0 \text{ m/s}\right)\left(\frac{1.53 \text{ s}}{2}\right) \\ &= 255 \text{ m} \end{aligned}$$

Example

Bats emit sounds with frequencies ranging from 14 kHz to over 100 kHz. If a particular sound wave has a frequency of 100 kHz and a speed of 344 m/s, what is the wavelength of this wave?

Solution

$$\begin{aligned} v = \lambda f \Rightarrow \lambda &= \frac{v}{f} \\ &= \frac{344 \text{ m/s}}{1.00 \times 10^{5} \text{ Hz}} \\ &= 3.44 \times 10^{-3} \text{ m} \\ &= 3.44 \text{ mm} \end{aligned}$$

The amount a sound wave scatters depends on the size of the object that the sound wave hits and the wavelength of the sound. The object has to be comparable to or larger than the wavelength in order for significant scattering (reflection) to occur. Otherwise, the wave will diffract around the obstacle and will not be affected very much by it.

Example

A bat emits a sound wave that enables it to echolocate an insect 0.90 mm in diameter. If the wavelength of the sound wave is equal to the diameter of the insect, what is the period of the sound emitted? Assume that the speed of sound is 344 m/s.

Solution

$$\begin{aligned} v = \lambda f \Rightarrow f &= \frac{v}{\lambda} \\ &= \frac{344 \text{ m/s}}{9.0 \times 10^{-4} \text{ m}} \\ &= 3.82 \times 10^{5} \text{ Hz} \\ T &= \frac{1}{f} \\ &= \frac{1}{3.82 \times 10^{5} \text{ Hz}} \\ &= 2.6 \times 10^{-6} \text{ s} \end{aligned}$$

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NOTES

ULTRASOUND

Ultrasonic imagery has become an important technology in medicine. Ultrasound is used to monitor the development of fetuses and collect information about organ function (e.g., the heart and the circulatory system). Ultrasonic imaging can be used in place of surgery to detect abnormalities and can also be used in physiotherapy to heat muscles.

Ultrasonic devices work by emitting high frequency sound pulses (1 to 5 MHz) and detecting their echoes or reflections. The probe, an instrument composed of several quartz crystals, produces ultrasonic pulses and receives the reflected echoes. Quartz is called a piezoelectric crystal because it has a very interesting property: if a mechanical stress is applied to the crystal, a voltage is generated. Also, if a voltage is applied across the crystal, its shape is deformed. A high frequency electrical signal applied to a quartz crystal will cause it to oscillate (undergo regular deformations). This crystal oscillation produces sound waves. The reflected echoes deform another quartz crystal, and the induced voltages produced can be analyzed electronically.

The echoes or reflections that occur at each boundary define an internal structure. The changes in the sound wave moving from a fluid to a soft tissue or from a soft tissue to a bone are examples of boundaries that will reflect ultrasonic pulses. These reflections are analyzed by a computer that is programmed to make sense of this data. The processed data is in the form of time delays (from which distances between the probe and boundary can be inferred) and echo intensities. Typically, millions of pulses are analyzed each second.

SEISMOLOGY

When stresses that build up in Earth's crust over time are suddenly released, the energy radiates away from the location of the earthquake as seismic waves. These seismic waves are classified as follows:

- Body waves—travel through the inner part of Earth
- Surface waves (L waves)—travel along the surface as a ripple and do most of the damage to man-made structures.

Body waves can be further subdivided into the given waves:

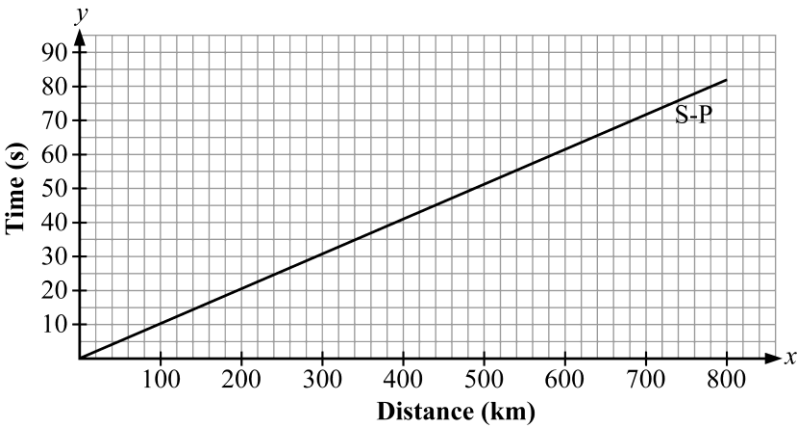
- Primary waves (P waves)—compression waves that travel 2 to 8 km/s. Much like sound waves, primary waves are longitudinal waves and are able to travel through solids, liquids, and gases.
- Secondary waves (S waves)—shear waves that push material perpendicular to the direction in which the waves travels. These waves travel slower than primary waves. S waves can only travel through solids because they are transverse waves. Consequently, Earth's core is a barrier to the propagation of S waves.

NOTES

Since primary waves travel approximately 1.7 times faster than secondary waves, seismologists can compute the distance from a recording station to the location of an earthquake. With similar information from two other recording stations, the precise location of an earthquake can be determined.

Example

A seismic recording station determines that the P waves reach the station 64 s ahead of the S waves. Use the graph to determine the distance from the station to the earthquake. Compare the speed of a P wave with the speed of sound through air at 15°C (about 340 m/s).



Solution

The distance corresponding to a time of 64 s can be read as a little less than 640 km.

The speed of a P wave is the reciprocal of the slope of the graph. Since the graph shows a straight line, the slope of the line must be constant, and therefore can be calculated using any value of intersection. In this case, the time of the P wave is found at a distance of 700 km.

$$\frac{1}{\text{slope}} = \frac{1}{\text{rise/run}}$$
$$= \frac{1}{(73\text{ s})/(700\text{ km})}$$
$$= 9.59\text{ km/s}$$

The speed of a P wave compared with the speed of sound through air can be calculated.

$$\frac{9.59\times10^{3}\text{ m/s}}{340\text{ m/s}} = 28$$

A P wave moves approximately 28 times faster than sound through air. This is not surprising since waves are expected to propagate faster through solids than through gases.

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Oil and gas tend to accumulate in porous rock, such as sandstone and limestone (called reservoir rock), underlying a non-porous rock, like granite or marble (called the cap rock). Seismology makes use of the fact that waves partially reflect from boundaries between these kinds of rock structures, and structures that look promising can be explored by test drilling. Seismic echoes also provide information about the porosity of the rock and even the type of fluid (water, oil, gas) in the rock. In exploration seismology, the source of the shock wave may be a heavy vehicle slamming a metal plate against the ground with a frequency around 10 Hz. Explosives can also be detonated along a predetermined path to produce a series of shock waves.

Example

A thumper truck is used to produce seismic shockwaves with a frequency of 25.0 Hz. Assuming the shockwave travels with an average speed of 7.50×10^3 m/s, how large would an oil reservoir have to be in order to be detected by the seismic crew? That is, what is the resolving capability of the shock wave?

Solution

$$v = \lambda f \Rightarrow \lambda = \frac{v}{f}$$
$$= \frac{7.50 \times 10^3 \text{ m/s}}{25.0 \text{ Hz}}$$
$$= 3.00 \times 10^2 \text{ m}$$

A small structure will require a smaller wavelength (higher frequency) wave in order to be detected.

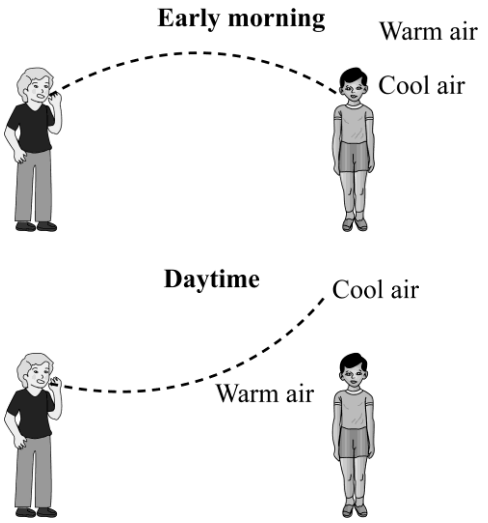
Infrasonic sound—frequency less than 20 Hz

Infrasonic sound is sound with a frequency less than 20 Hz. Elephants, whales, and rhinoceros are believed to communicate by producing very low frequencies. Other sources of infrasonic sound include avalanches, earthquakes, and thunder.

NOTES

REFRACTION OF SOUND

Sound has been observed to travel farther in the early morning than later in the day. This difference in travel distance is because the morning air is cooler at Earth's surface, and later in the day the air is usually warmer. A sound wave speeds up as it passes through warmer air, causing the sound to deflect toward cooler air, as shown in the following diagrams.



INTERFERENCE OF SOUND

If you have ever connected loudspeakers to a sound system, you may have observed the electrical terminals are colour coded, usually red and black, or labelled, usually + and –. The loudspeakers must be connected so they move in phase when a sound signal reaches them. This ensures that both speaker cones are moving in and out at the same time. The compression formed by the left speaker cone moving outward will constructively reinforce the compression formed by the right speaker cone moving outward. This behaviour will improve the bass response immensely.

If the loudspeaker leads are incorrectly connected, the left speaker cone moves outward as the right speaker cone moves inward and vice versa. Destructive interference occurs, and the bass response of the system is weakened.

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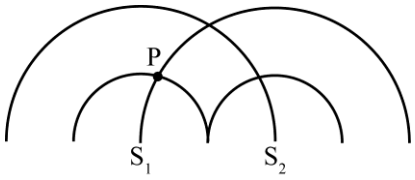
NOTES

Example

Two identical loudspeakers, S_1 and S_2, are in phase and responding to an input signal of 30.0 Hz. If a student with a sound pressure meter moves in front of S_1 until a maximum reading is observed and the distance to S_1 is measured to be 11.3 m when this maximum occurs, what is the closest possible distance between the student and S_2? Assume sound travels at 340 m/s.

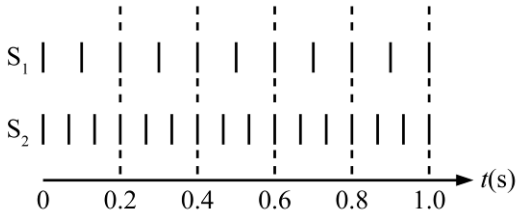
Solution



The student is located a distance of one wavelength from S_1 and is at a point of constructive interference. Thus, the crest from S_1 is meeting a crest from S_2. Since the student is in front of S_1, and not equidistant from both points, he can be no closer than two wavelengths from S_2 (point P in the diagram). The distance from P to S_2 must therefore be double the distance from P to S_1 $\left(11.3\text{ m}\times 2 = 22.6\text{ m}\right)$.

BEATS

Consider two sources of sound that are in phase. Source 1 and source 2 emit sinusoidal waveforms with frequencies of 10.0 Hz and 15.0 Hz, respectively. In the diagram below, the straight line segments represent the compressions in each wave.



Two wavelengths coming from S_1 are equivalent to three wavelengths coming from S_2. Thus, the two waves start in phase, drift out of phase and come into phase again every 0.2 s, as indicated by the dashed line. The constructive interference occurring with a period of 0.2 s is called a beat.

NOTES

Beats result from the interference of two or more sound waves.

Sound waves with frequencies of 10.0 Hz and 15.0 Hz are not audible, but if higher frequency sound waves were used, the beats would be heard as a regular variation in loudness. The number of beats per second is the number of times the waves move into phase with each other per second. If you know the frequencies of the two sources, you can calculate the number of beats by subtracting their frequencies.

$$\text{Beat frequency} = f_b = f_2 - f_1$$

Example

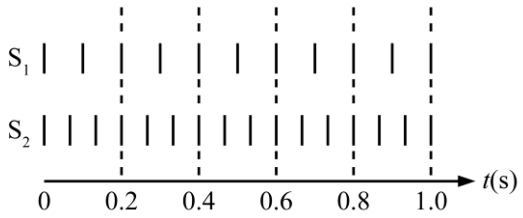
When a tuning fork of unknown frequency is struck simultaneously with a tuning fork with a frequency of 540 Hz, 16.0 beats are heard in 4.00 s. Assuming that sound travels at 340 m/s, what are two possible frequencies for the unknown tuning fork?

Solution

$$f_b = \frac{16.0 \text{ beats}}{4.00 \text{ s}}$$
$$= 4.000 \text{ Hz}$$
$$\therefore f_2 - f_1 = f_b$$
$$= 4.000 \text{ Hz}$$
$$f_2 = f_1 \pm 4.000 \text{ Hz}$$
$$= 540 \text{ Hz} \pm 4.000 \text{ Hz}$$
$$= 536 \text{ Hz and } 544 \text{ Hz}$$

Example

At what times in the given diagram should destructive interference occur?



Solution

Destructive interference occurs when compressions from one source line up with rarefactions from the other source. This occurs at $t = 0.1$ s, $t = 0.3$ s, $t = 0.5$ s, $t = 0.7$ s, and $t = 0.9$ s.

SHOCK WAVES

When an object travels through a medium faster than sound can travel through the same medium, a shock wave is produced. A similar effect involving water waves can be observed when boating. When the boat is drifting slowly, the disturbance it produces in the water travels through the water faster than the boat. However, when the boat accelerates, it quickly overtakes these water waves and begins to form a V-shaped bow wave.

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The shock wave produced by the boat in the water has the same basic feature as the shock wave produced by a plane flying faster than the speed of sound. When the shock wave produced by a plane exceeding the speed of sound passes overhead, a sonic boom is heard. The shock wave forms at Mach 1, and it has the shape of a cone. This cone is more slender at Mach 2.

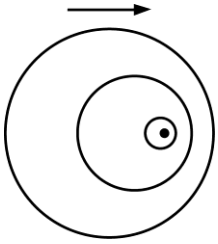
NOTES

Mach 1 = the speed of sound

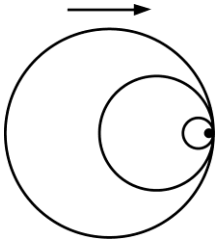
Mach 2 = double the speed of sound

When a whip is cracked, a tiny sonic boom is heard because the tip of the whip is moving through the air faster than sound.

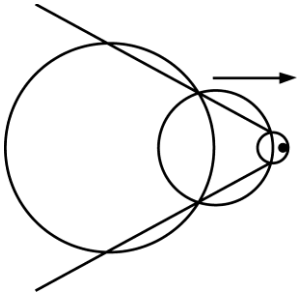
A shock wave is an example of interference of sound waves. The following diagrams offer further explanation.



The object (plane) is travelling slower than sound. There is no interference of the waves.



The object (plane) is travelling at the speed of sound. Although there is a large compression in front of the object, this compression does not reach the ground. A great deal of energy is required to break through this compression.



The object (plane) is travelling faster than sound. The waves interfere to produce a maximum compression that spreads away from the object. When this high compression passes overhead, you quickly go from an area of high compression (constructive interference) to an area of low compression, leading to the creation of a sonic boom or shock wave.

NOTES

Example

What is the speed of sound in air at 25.0°C?

Solution

$$\begin{aligned} v &= \left(331 + \left(0.59/°\text{C}\right)T\right) \text{ m/s} \\ &= \left(331 + \left(0.59/°\text{C}\right)\left(25.0°\text{C}\right)\right) \text{ m/s} \\ &= 346 \text{ m/s} \end{aligned}$$

Example

If $v_{\text{sound}} = 335 \text{ m/s},$ how long does it take sound to travel 2.0 km through air?

Solution

$$\begin{aligned} v &= \frac{d}{t} \\ t &= \frac{d}{v} \\ &= \frac{2.0 \times 10^3 \text{ m}}{335 \text{ m/s}} \\ &= 6.0 \text{ s} \end{aligned}$$

Example

A tuning fork with a frequency of 256 Hz is sounded. If $v_{\text{sound}} = 341 \text{ m/s},$ what is the distance between successive compressions of the sound wave produced by this tuning fork?

Solution

$$\begin{aligned} v &= \lambda f \\ \lambda &= \frac{v}{f} \\ &= \frac{341 \text{ m/s}}{256 \text{ Hz}} \\ &= 1.33 \text{ m} \end{aligned}$$

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PRACTICE EXERCISES

Formulas: $v = \frac{d}{t}$ $v = \lambda f$ $T = \frac{1}{f}$

$v = \left(331 + \left(0.59/°\text{C}\right)T\right)$ m/s Beat frequency = $f_b = f_1 - f_2$

1. What is the speed of sound at 10.0°C in dry air?

Use the following information to answer the next question.

Two tuning forks vibrate with frequencies of 256 Hz and 251 Hz.

2. If these two forks are sounded together, how many beats per second are heard?

Use the following information to answer the next question.

Taking the average speed of sound through air to be 341 m/s, a student observed that the time delay between a lightning strike and the resulting thunder was 5.50 s.

3. How far away from the student did the lightning strike?

Use the following information to answer the next question.

A key on a piano produces a wave with a frequency of 27.5 Hz.

4. If $v_{sound} = 343$ m/s, what is the distance between successive compressions in the sound wave as it travels through air?

5. When the flash from a gun is observed 3.6 s before the sound is heard, how far is the gun from the observer? (T = 22°C)

6. If a sonar signal reflected from the bottom of a lake is detected 0.62 s after the original signal was transmitted and $v_{sound} = 1.46 \times 10^3$ m/s, how deep is the lake?

7. If a sound wave travels through air that is at 18.0°C, how long does it take for the wave to travel 10.0 km?

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Use the following information to answer the next question.

A bat produces an ultrasonic sound with a frequency of 2.50×10^5 Hz.

8. If it is the surrounding air is at 20.0°C what is the distance between successive compressions of the wave?

9. At what temperature will the speed of sound through air be 355 m/s?

Use the following information to answer the next question.

A piano string and a tuning fork are sounded at the same time.

10. If the frequency of the tuning fork is 6.60×10^2 Hz and 3.00 beats per second are observed between the two waves, what is the frequency of the piano string?

Use the following information to answer the next question.

A sound wave with a frequency of 1.00×10^3 Hz travels through water.

11. If the compressions of this wave are 1.5 m apart, what is its speed?

Use the following information to answer the next question.

While fishing near a high cliff, a fisherman shouts.

12. If he hears the echo 1.05 s after he shouts and the temperature of the surrounding air is 17.0°C, how far is he from the cliff?

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Use the following information to answer the next question.

A tuning fork vibrates with a frequency of 256 Hz.

13. What is its period of vibration?

Use the following information to answer the next question.

A dolphin emits an ultrasonic sound under water that has a period of 2.0×10^{-5} s.

14. If the successive compressions are 3.0×10^{-2} m apart, what is the speed of the sound through the water?

Lesson 4 CHARACTERISTICS OF SOUND

NOTES

The three characteristics of sound are pitch, loudness, and quality.

The frequency of a sound is described in terms of its **pitch**. A high-pitched sound corresponds to a high frequency. A low-pitched sound corresponds to a low frequency. Pitch can be related to musical notes on a scale. There is limit to the range of frequencies that humans can hear. This limit is called the audible range, and it is generally between 20 Hz and 20 000 Hz.

The **loudness** of sound is related to the energy of the wave. This energy is associated with the wave's amplitude (the degree of compression or rarefaction). Loudness is a subjective measure that varies from person to person and depends upon both the frequency of the sound and its duration.

Intensity decreases as the inverse square of the distance.

$$I \propto \frac{1}{d^2}$$

A much more objective measure is **sound intensity**. Intensity is the amount of sound energy passing through a unit area each second. If one joule of energy passes through an area of one square metre in one second, the intensity of this sound is $1\ \frac{\text{J/s}}{\text{m}^2} = 1\ \frac{\text{W}}{\text{m}^2}$. A sound intensity of one watt per square metre is a very loud sound.

The order of magnitude distance between Vancouver and Toronto is 10^6 m.

$$I \propto \frac{1}{d^2} \Rightarrow \frac{1}{\left(10^6\right)^2}$$

$$\Rightarrow \frac{1}{10^{12}} = 10^{-12}$$

The human ear is able to respond to an amazing range of sound intensities—a range with an order of magnitude of 10^{12}. To better illustrate this vast difference, envision a 50 watt light bulb situated at the heart of Vancouver. The difference in the intensity range of the human ear is analogous to measuring the intensity of the light bulb right next to the bulb and measuring the intensity of the bulb in Toronto.

This range is so large that a logarithmic scale was defined to compress it into a more usable format. The decibel (dB), one tenth of a bel (B), is the standard unit used for sound intensity level (L). It is defined as follows:

$$L = 10\log_{10}\left(\frac{I_1}{I_0}\right)\text{dB}$$

This relationship means that the decibel compares any two intensities of sound or sound pressure levels, I_1 and I_0. For example, if the sound pressure level of one sound is 10 times larger than the sound pressure level of another sound, the larger sound will have a magnitude of 10 dB.

$$\begin{aligned} dB &= 10\log_{10}(10) \\ &= 10(1) \\ &= 10 \end{aligned}$$

- 10 dB corresponds to a pressure level 10^1 times larger
- 20 dB corresponds to a pressure level 10^2 times larger
- 30 dB corresponds to a pressure level 10^3 times larger
- 100 dB corresponds to a pressure level 10^{10} times larger

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NOTES

I_0 is generally defined as the minimum intensity required for a sound to be heard by a good human ear: $I_0 = 1.0 \times 10^{-12}$ W/m^2. Using I_0 as the reference intensity, the human ear is capable of responding to sound intensity levels ranging from about 0–120 dB.

Many daily events can have significantly different decibel measures compared to the threshold of hearing:

- Rock concert 120 dB
- School cafeteria at lunch 85 dB
- Busy street traffic 70 dB
- Classroom 50–60 dB
- Whisper at 1.0 m 20 dB

Highway traffic noise is a dominant noise in many environments. In order to reduce the effect of this noise, noise barriers are built. These barriers may be constructed using dirt mounds, vertical walls, or a combination of dirt mounds and vertical walls. Noise barriers reduce the noise by absorption and reflection, but they only reduce the intensity of the sound by about 5–10 dB.

QUALITY

The quality of sound depends on the number of harmonics present and the amplitude of the sound waves. The same musical note played on two different instruments does not sound the same because of the difference in their quality.

Wave Shape	Instrument That Creates Wave
	Tuning fork
	Organ pipe
	Clarinet
	Violin

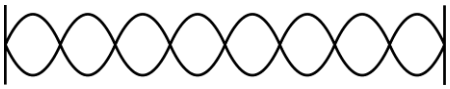
NOTES

The characteristics of sound can be demonstrated using an oscilloscope and a tape recorder. Even though sound is a longitudinal wave, an oscilloscope shows the sound wave on its screen as a transverse wave. Using a tape recorder as an amplifier, a sound signal can be input into the oscilloscope and the following observations can be made:

- Changing the pitch of the sound will result in waveform changes. High pitch is associated with short wavelengths (high frequency).
- Changing the loudness of a sound will change the amplitude of the waveform.
- The signals coming from different musical instruments have different waveforms that clearly show the quality of the sound.

INTERFERENCE AND MUSICAL INSTRUMENTS

An important application of wave interference occurs in musical instruments. When the stretched string of a string instrument is plucked or the air column in a wind instrument is set in motion by a vibrating object, like a reed or human lips, standing waves are created. **Standing waves** are produced by interference. The term standing wave comes from the fact that the wave pattern does not move.

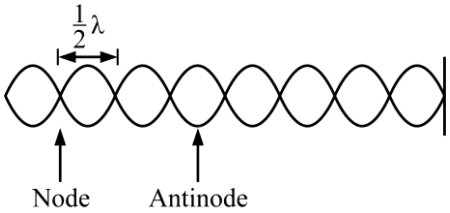


A standing wave results when two wave trains of equal wavelength and amplitude travel through the same medium in opposite directions. This can happen when a wave train is reflected from a fixed object.

Standing waves have alternating nodes and antinodes.

- Node—region of where there is zero amplitude
- Antinode—region where there is maximum amplitude

The distance between nodes is $\frac{1}{2}\lambda$.



Standing waves are produced by interference between the reflected and incident waves.

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NOTES

Both a pendulum and a mass vibrating at the end of a spring have one frequency at which they vibrate; a stretched string or air column will vibrate at a number of frequencies. The lowest frequency of vibration is called the fundamental frequency, or first harmonic. All the other frequencies at which the string or air column vibrates are whole number multiples of the fundamental frequency.

- f = fundamental frequency (or first harmonic)
- $2f$ = second harmonic
- $3f$ = third harmonic
- etc.

The harmonics on a stretched string or air column can exist simultaneously and superpose, resulting in very complex wave forms. Different musical instruments produce unique harmonics that combine in very distinctive ways. It is these different harmonic combinations that give different musical instruments their characteristic sounds.

MECHANICAL RESONANCE

If you are pushing a child on a swing, there is a right time to push and a wrong time to push. If you push at the right time, the child will swing higher. If you push at the wrong time, the child will not swing higher. The natural frequency of vibration of the child-swing system is determined by the length of the swing and if energy is supplied to this system with a frequency matching the system's natural resonant frequency. Even small energy inputs at the natural frequency can result in large amplitude oscillations.

$$f = \frac{1}{T}$$

All oscillating systems have natural resonant frequencies. Atoms, molecules, mechanical systems, and electric circuits all have resonant frequencies.

Mechanical resonance occurs when two objects have the same natural frequency and one of them is set to vibrate. In a short time, the second object will also start to vibrate.

You can demonstrate mechanical resonance by tying two simple pendulums of the same length to a tightly stretched cord.

Sometimes resonance can be undesirable or even destructive.
For example, a car may begin to vibrate at a specific speed. Perhaps an unbalanced tire causes a tiny oscillation that matches the natural resonant frequency of the entire car body at a particular speed. The oscillations can be eliminated by either increasing or decreasing the speed of the car.

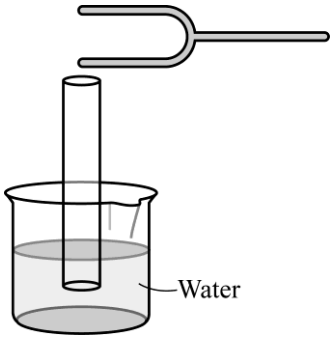
The resonance caused by gusts of wind was responsible for the collapse of the Tacoma Narrows Bridge in 1940. Soldiers marching in step have also been known to create resonance in bridges strong enough to collapse them. It is now a rule that when soldiers cross a bridge, they break step. There is also the story of an opera singer shattering a glass goblet by hitting a high musical note, but this would require his or her voice to be amplified.

NOTES

ACOUSTICAL RESONANCE

Acoustical resonance can be demonstrated by holding a vibrating tuning fork over a closed air column, as demonstrated in the diagram below.

Resonance and standing waves are important to music.



The length of the air column can be adjusted to create standing waves in the column as a result of resonance.

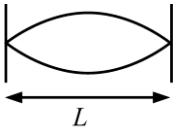
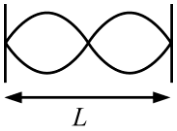
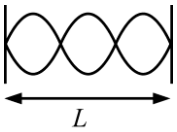
Resonance and standing waves are important to music. Musical instruments are vibrating systems. The vibrating element is either a stretched string or an air column. A string is set into vibration by plucking, strumming, or drawing a bow across it. An air column is put into vibration by a vibrating reed or the musician's lips. Musical instruments are classified as stringed instruments and wind instruments.

In the case of stringed instruments, the source of the sound is the vibrating string; however, these strings are attached to a resonating box, such as the body of a guitar, or a resonating surface, such as the soundboard of a piano. This box or surface serves to amplify the sound. Acoustical resonance is no different than mechanical resonance. The natural resonant frequencies of the string depend on its tension, mass, and length.

The formula for the fundamental frequency of a stretched string is $\lambda = 2L$.

STRETCHED STRINGS

The vibrating string in any string instrument is attached at both ends. Therefore, the standing wave produced on this string must have nodes at the fixed ends. The following diagrams show the simplest examples of a standing wave pattern.

$L = \frac{\lambda}{2}$ (or $\lambda = 2L$)		Fundamental frequency (first harmonic)
$\lambda = L$		Second harmonic
$L = \frac{3\lambda}{2}$		Third harmonic

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NOTES

A stretched string can vibrate at all harmonics (first, second, third, fourth, etc.) simultaneously.

The wavelength of the second harmonic is $\frac{1}{2}$ the wavelength of the first harmonic, and the frequency of the second harmonic is two times greater.

The wavelength of the third harmonic is $\frac{1}{3}$ the wavelength of the first harmonic, and the frequency of the third harmonic is three times greater. For the case of a string attached at both ends, this can be simplified to $L = \frac{n\lambda}{2}$.

A standing wave has a wavelength that is a function of L (length of string), as shown above.

The natural frequency of a vibrating string also depends on its tension and mass.

The speed of vibration through a string depends on the tension and the mass per unit length:

$$v = \sqrt{\frac{T}{m/L}}$$

where

v = speed
T = tension
m = mass
L = length

This equation is similar to the velocity equation in the following form:

$$v = \sqrt{\frac{\text{elasticity}}{\text{density}}}$$

NOTES

Example

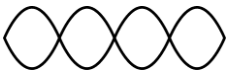
A 0.60 m string vibrates at a fundamental frequency of 125 Hz. What is the speed of the wave through the string?

Solution

$$\begin{aligned}\lambda &= 2L \\ &= 2(0.60\text{ m}) \\ &= 1.20\text{ m} \\ v &= \lambda f \\ &= (1.20\text{ m})(125\text{ Hz}) \\ &= 1.5\times10^{2}\text{ m/s}\end{aligned}$$

Example

A vibrating string has a frequency of 375 Hz. If this string is vibrating in four segments, what is its fundamental frequency?



The harmonic is one less than the number of nodes. We know this is the fourth harmonic, because there are five nodes.

Solution

$$\begin{aligned}\text{Fundamental } f &= \frac{375\text{ Hz}}{4} \\ &= 93.8\text{ Hz}\end{aligned}$$

Example

A 1.2 m string has a mass of 1.9 g. This string is tightened to a tension of 18.9 N. What is the speed of a wave travelling on this string?

Solution

$$\begin{aligned}v &= \sqrt{\frac{T}{m/L}} \\ &= \sqrt{\frac{18.9\text{ N}}{(1.9\times10^{-3}\text{ kg})/(1.2\text{ m})}} \\ &= 1.1\times10^{2}\text{ m/s}\end{aligned}$$

Unit analysis:

$$\begin{aligned}\sqrt{\frac{\text{N}}{\text{kg/m}}} &= \sqrt{\frac{\text{kg}\cdot\text{m/s}^2}{\text{kg/m}}} \\ &= \sqrt{\text{m}^2/\text{s}^2} \\ &= \text{m/s}\end{aligned}$$

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ACTIVITY 3

Purpose

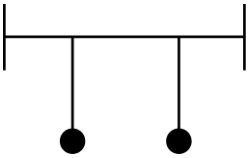
To examine the nature of mechanical resonance.

Apparatus

Two equal masses
Several lengths of string

Procedure

1. Attach two pendulums of the same length (same period) to a tightly stretched cord, as shown in the diagram.



2. Start one of the pendulums vibrating.
3. Wait a few seconds, and the other pendulum will begin to vibrate with increasing amplitude.

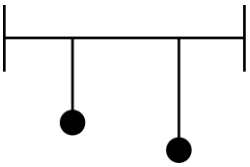
Question

Why does the second pendulum start to vibrate?

Think back to the example of the child on a playground swing. In order for the child to swing higher, he or she must be pushed at the right time. The swing has a natural resonant frequency, and energy input at this frequency is absorbed completely.

This happens to the second pendulum. The energy of the vibration of the first pendulum is transferred through the cord, supplying a force to the second pendulum at just the right time.

Now, try this:
4. Attach two pendulums of different lengths (different period) to a tightly stretched cord, as shown in the diagram.



5. Start one of the pendulums vibrating.

Question

What happens to the second pendulum?

It does not begin to vibrate with increasing amplitude.

ACTIVITY 4

Purpose

To analyze the effects of acoustic resonance.

Apparatus

Two tuning forks of the same frequency

Procedure

1. Place each tuning fork on the opposite side of a desk.
2. Strike the tine of one tuning fork with a rubber mallet.
3. Wait a few seconds, then eliminate the vibration of this tuning fork.
4. Observe the second fork. It should be vibrating.

Question

How can what you have just observed be explained?

The energy of the wave produced by the first fork supplied force at the right time to make the second fork vibrate.

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PRACTICE EXERCISES

Formulas: $v = \lambda f$ $v = \sqrt{\frac{T}{m/L}}$ $L = \frac{n\lambda}{2}$

Use the following information to answer the next question.

A 1.2 m string vibrates at a fundamental frequency of 65 Hz.

1. What is the speed of the wave through the string?

Use the following information to answer the next question.

A vibrating string has a frequency of 144 Hz.

2. If this string is vibrating in three segments, what is its fundamental frequency?

3. If the fundamental frequency of a vibrating string is 354 Hz, what is the frequency of its third harmonic?

4. If the frequency of the fourth harmonic of a vibrating string is 2.5×10^3 Hz, what is the fundamental frequency of this string?

5. If the frequency of the third harmonic of a vibrating string is 1.2×10^3 Hz, what is the frequency of its second harmonic?

Use the following information to answer the next question.

A string on a musical instrument vibrates in three segments at a frequency of 335 Hz.

6. If this same string vibrates in four segments, what is its frequency?

Use the following information to answer the next question.

The fundamental frequency of a 27.0 cm string is 637 Hz.

7. What is the fundamental frequency of this string if its length is 22.0 cm long?

Use the following information to answer the next question.

A string is 25 cm long and has a fundamental frequency of 441 Hz.

8. In order to produce a fundamental frequency of 525 Hz, to what length must the string be shortened?

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Use the following information to answer the next question.

A 0.80 m string has a mass of 1.9 g. This string is tightened to a tension of 23 N.

9. What is the speed of a wave travelling on this string?

Use the following information to answer the next question.

A 0.80 m string has a mass of 1.9 g. This string is tightened to a tension of 23 N.

10. What is the fundamental frequency of the sound produced by this vibrating string?

Use the following information to answer the next question.

A 5.0 m cord has a mass of 195 g. The cord is tightened to a tension of 80.0 N.

11. What is the fundamental frequency of the vibrating cord?

Use the following information to answer the next question.

A 2.6 m cord has a mass of 98 g.

12. In order for its fundamental frequency of vibration to be 133 Hz, to what tension must the cord be tightened?

13. If a 0.600 m string under a tension of 1.00×10^2 N produces a fundamental frequency of 224 Hz, what tension would be required to produce a fundamental frequency of 231 Hz?

Use the following information to answer the next question.

A 0.50 m string is tightened to a tension of 12 N.

14. If the speed of the wave travelling in this string is 65 m/s, what is the mass of the string?

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Lesson 5 AIR COLUMNS

NOTES

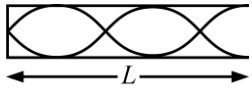
In the case of wind instruments, the reed or human lips vibrate at all frequencies; however, the resonating air column can only resonate at certain frequencies.

There are two types of air columns:

- Closed air columns
- Opened air columns

CLOSED AIR COLUMNS

For a standing wave in a closed air column, there is always a node at the closed end and an antinode at the open end.

	Fundamental frequency (first harmonic)	$L = \frac{\lambda}{4}$ (or $\lambda = 4L$)
	Third harmonic	$L = \frac{3}{4}\lambda$
	Fifth harmonic	$L = \frac{5}{4}\lambda$

$$\lambda = \frac{4L}{3}$$

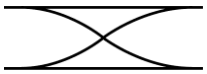
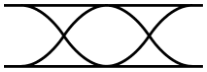
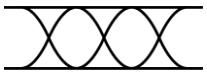
$$\lambda = \frac{4L}{5}$$

Air columns in closed tubes will resonate (vibrate) at odd numbered harmonics only: first, third, fifth, seventh, etc.

The wavelength of the third harmonic is $\frac{1}{3}$ that of the first harmonic; the wavelength of the fifth harmonic is $\frac{1}{5}$ that of the first harmonic.

OPENED AIR COLUMNS

For a standing wave in an opened air column, there is always an antinode at both ends.

	Fundamental frequency (first harmonic)	$L = \frac{\lambda}{2}$ (or $\lambda = 2L$)
	Second harmonic	$L = \lambda$
	Third harmonic	$L = \frac{3\lambda}{2}$

Compare $\lambda = f(L)$ for vibrating strings and for open-air columns.

An opened tube will resonate at all harmonics: first, second, third, etc.

NOTES

Example

A closed air column resonates with a fundamental frequency of 225 Hz. If the speed of the sound wave is 341 m/s, what is the length of this air column?

Solution

$$v = \lambda f$$
$$\lambda = \frac{v}{f}$$
$$= \frac{341 \text{ m/s}}{225 \text{ Hz}}$$
$$= 1.516 \text{ m}$$
$$\lambda = 4L$$
$$L = \frac{\lambda}{4}$$
$$= \frac{1.516 \text{ m}}{4}$$
$$= 0.379 \text{ m}$$

Example

An opened air column resonates with a fundamental frequency of 334 Hz. If the length of the air column is 51.0 cm, what is the speed of the sound wave?

Solution

$$\lambda = 2L$$
$$= 2(0.510 \text{ m})$$
$$= 1.020 \text{ m}$$
$$v = \lambda f$$
$$= (1.020 \text{ m})(334 \text{ Hz})$$
$$= 341 \text{ m/s}$$

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ACTIVITY 5

Purpose

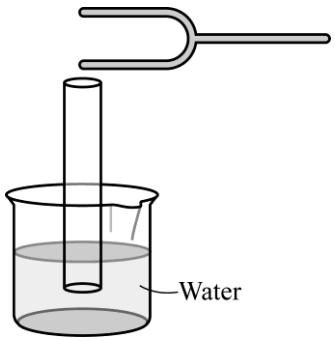
To determine the speed of sound

Apparatus

- An open tube at least 0.4 m long
- Tuning fork of known frequency
- Tall glass cylinder
- Water

Procedure

1. Set up an apparatus, as shown in the given diagram.



2. Set the tuning fork vibrating.
3. Pull the open tube upward slowly while the fork is vibrating. Listen carefully until you hear the resonance in the tube (the sound will get louder).
4. Mark the water level on the tube. Remove the tube from the water, and measure from the top of the tube to the level just marked. This is the length of the closed air column.

Data

- Frequency of the tuning fork:
- Length of closed air column:

Calculation

1. Calculate the speed of sound from the data above.

2. Compare this calculated value with the theoretical value.

If you were unable to collect data for this activity, use the following data to calculate the speed of sound.

- Frequency of tuning fork: 256 Hz
- Length of closed air column: 1.34 m

PRACTICE EXERCISES

Formulas: $v = \lambda f$

Fundamental frequencies
– closed tubes: $\lambda = 4L$
– opened tubes: $\lambda = 2L$

Use the following information to answer the next question.

A closed air column resonates with a fundamental frequency of 256 Hz.

1. If the speed of the sound wave is 343 m/s, what is the wavelength of the sound at the fundamental frequency?

Use the following information to answer the next question.

A closed air column resonates with a fundamental frequency of 4.40×10^2 Hz.

2. If the air column is 18.9 cm long, what is the speed of the sound wave?

3. If a closed air column resonates with a fundamental frequency of 384 Hz, what is the frequency of its third harmonic?

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4. What will be the fundamental frequency of a 22.0 cm long closed air column with an air temperature of 20.0°C?

5. What is the length of a closed air column that produces a fundamental frequency of 256 Hz at 20.0°C?

Use the following information to answer the next question.

A 55 cm closed air column will resonate at many frequencies. However, the lowest frequency at which it resonates is 156 Hz.

6. What is the speed of the sound wave?

7. Assuming that the speed of sound is 341 m/s, what is the shortest closed air column that will resonate to a sound source with a frequency of 4.40×10^2 Hz?

Use the following information to answer the next question.

An opened air column resonates with a fundamental frequency of 256 Hz.

8. If the speed of the sound wave is 341 m/s, what is the length of the air column?

Use the following information to answer the next question.

An opened air column resonates with a fundamental frequency of 512 Hz.

9. If the length of the air column is 33.0 cm, what is the speed of the sound wave?

10. If an opened air column resonates with a fundamental frequency of 384 Hz, what is the frequency of the next greatest harmonic with which this column will resonate?

11. If a closed air column resonates with a fundamental frequency of 384 Hz, what is the frequency of the next greatest harmonic with which this column will resonate?

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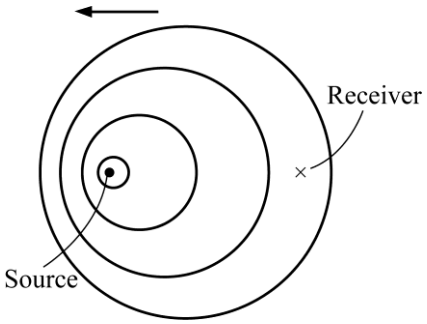
Lesson 6 DOPPLER EFFECT

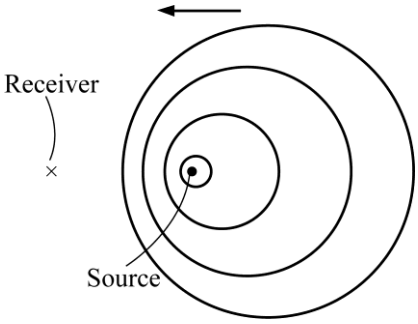
NOTES

If there is a relative motion between the source and the receiver of the wave, the frequency received will be different from the source's frequency.

If the source is moving away from the receiver, the frequency received will be less than the frequency of the source.



If the source is moving toward the receiver, the frequency received will be greater than the frequency of the source.



This is known as the Doppler effect. Have you ever been waiting at a railroad crossing for a train to go by? As the train approaches with its whistle blowing, you hear a high frequency (pitch) sound. When the train (source of the whistle) has passed, the sound drops to a low frequency (pitch).

PRACTICE EXERCISES

Use the following information to answer the next two questions.

A car is travelling toward a stationary sound (a whistle) that has a frequency of 625 Hz.

1. Is the apparent frequency of the sound, as heard by the driver of the car, higher or lower than 625 Hz?

2. If the car in question number one was travelling away from the stationary sound, is the apparent frequency of the sound, as heard by the driver of the car higher, or lower than 625 Hz?

Use the following information to answer the next question.

A whistle with a frequency of 965 Hz is moving toward a stationary observer.

3. Is the apparent frequency of the sound, as heard by the observer, higher or lower than 965 Hz?

4. If the whistle in question number three was moving away from the stationary observer, is the apparent frequency of the sound, as heard by the observer, higher or lower than 965 Hz?

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PRACTICE TEST

Use the following information to answer the next question.

A train is travelling at a speed of 80 km/h. An observer hears its whistle blowing as he stands near the railroad track, and the frequency of the whistle sound appears to rise (increase) as the train approaches.

1. This phenomenon is referred to as

A. refraction

B. diffraction

C. the Doppler effect

D. the superposition principle

Use the following information to answer the next question.

A transverse wave is travelling through a medium at a speed of 10.0 cm/s.

2. If its amplitude is 5.0 cm and its wavelength is 2.0 cm, what is the frequency of the wave?

3. In an interference pattern, there are maxima and minima. The maxima result when waves meet

A. in phase

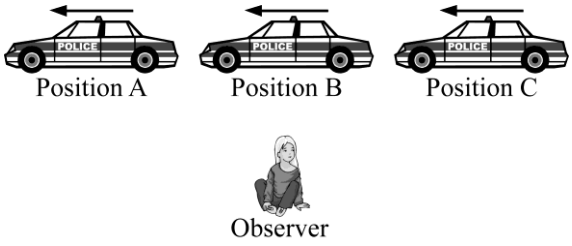
B. out of phase

C. with the same amplitudes

D. with the same frequencies

Use the following information to answer the next question.

A police car is sounding its siren and is travelling at 100 km/h as it goes by. An observer is sitting along side the road, as shown in the diagram.



4. At which position would the frequency of the siren appear to be the lowest to the observer?

A. Position A

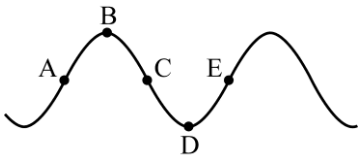
B. Position B

C. Position C

D. It would appear to be the same at all positions.

Use the following information to answer the next question.

The given diagram represents a transverse wave.



5. A wavelength can best be described as the distance between points

A. A and C

B. B and D

C. A and E

D. C and E

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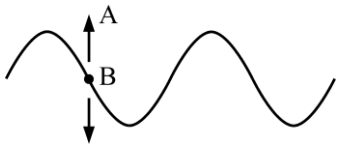
Use the following information to answer the next question.

A particle in a mechanical wave moves from point B to point A.



6. Which of the following statements concerning the speed of this particle is **true**?

A. The particle is moving faster at point A.

B. The particle is moving faster at point B.

C. The particle has the same speed at points A and B.

D. This is a standing wave so the particle does not move.

Use the following information to answer the next question.

A 4.5 m rope is vibrating in such a way as to produce a standing wave as shown in the given diagram.

7. If the frequency of the vibration is 60.0 Hz, what is the speed of the wave as it travels through the rope?

8. The Doppler effect occurs when

A. there is a path difference of 0.5λ between two waves

B. there is relative motion between a wave source and the observer

C. side-by-side tuning forks are emitting slightly different frequencies

D. two objects have the same natural frequency, and the vibration of one of the objects causes the other object to vibrate

9. The phenomenon of resonance occurs when

A. there is a path difference of 0.5λ between two waves

B. there is relative motion between a wave source and the observer

C. side-by-side tuning forks are emitting slightly different frequencies

D. two objects have the same natural frequency, and the vibration of one of the objects causes the other object to vibrate

10. Energy can travel as waves or as particles. Which of the following characteristics of waves is shared with particles?

A. Refraction

B. Diffraction

C. Destructive interference

D. Constructive interference

Use the following information to answer the next question.

A mass is attached to a horizontal spring on a frictionless table. The mass is pulled 10 cm to the right and then released.

11. The amplitude of the resulting oscillation is

A. 20 cm

B. 15 cm

C. 10 cm

D. 5 cm

Use the following information to answer the next question.

A sound wave has a frequency of 10.2 kHz and a wavelength of 49.1 cm when travelling through iron.

12. How long will it take this wave to travel the length of a 13.0 m iron beam?

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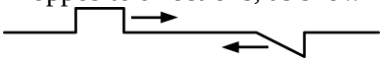
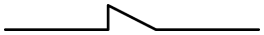
Use the following information to answer the next question.

Two idealized pulses are travelling in opposite directions, as shown in the diagram.



13. Which of the following diagrams depicts the shape of the superimposed wave?

A.



B.

C.

D.

Use the following information to answer the next question.

The frequency of red light is constant whether the light is passing through air, water, glass, etc. Red light has a wavelength of 7.0×10^{-7} m and travels with a speed of 3.00×10^{8} m/s in air. When it enters glass, its speed is reduced to 2.0×10^{8} m/s.

14. What is the wavelength of red light in glass?

Use the following information to answer the next question.

Two guitar strings are plucked. One vibrates with a frequency of 440 Hz; the other vibrates with a frequency of 435 Hz.

15. How many beats will be heard in 5.0 s?

Use the following information to answer the next question.

A student has two identical tuning forks, each with a natural frequency of 525 Hz. He tapes a small washer to each of the tines of one of the forks and sounds both tuning forks together. The student hears 20.0 beats in 8.0 s. The pitch of the loaded tuning fork is audibly lower.

16. Assuming the speed of sound is 342 m/s, what is the wavelength of the loaded tuning fork?

17. Which property allows a listener to distinguish the sounds produced by a trumpet and a trombone, even when both instruments play the same note with the same loudness?

A. Amplitude

B. Frequency

C. Quality

D. Pitch

18. Which of the following frequencies is infrasonic?

A. 12 Hz

B. 60 Hz

C. 5 000 Hz

D. 16 000 Hz

Use the following information to answer the next question.

A projectile is launched from a level plane with a horizontal speed of 1 250 m/s. An observer (on the same level plane) sees the projectile directly overhead 2.00 s before he hears the sound of the explosion that launched the projectile.

19. If the speed of sound is 340 m/s, how far is the observer from the launcher?

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Use the following information to answer the next question.

A 9.5 m long rope contains five nodes.

20. If it vibrates with a frequency of 4.5 Hz, what is the speed of the wave?

21. When the temperature is 23°C, what is the length of a closed air column that is required to resonate with a fundamental frequency of 350 Hz?

Use the following information to answer the next question.

A ship's whistle is a 1.35 m tube, which is closed at one end. Assume the speed of sound is 343 m/s

22. Calculate the two lowest resonant frequencies for the whistle.

Use the following information to answer the next question.

A sound wave with a wavelength of 145 cm sets up a standing wave in a tube with a moveable piston, which closes one end. The piston is moved, making the air column progressively longer.

23. What are the first three resonant lengths of the tube?

A. 36.3cm, 109 cm, and 182 cm

B. 72.5 cm, 145 cm, and 218 cm

C. 72.5 cm, 218 cm, and 363 cm

D. 145 cm, 290 cm, and 435 cm

Use the following information to answer the next question.

An open tube is 1.35 m long.

24. If the speed of sound is 343 m/s, what is the resonant frequency of the first harmonic?

25. When a trumpet is played, the bell end (the end that flares open as a cone) is expected to be

A. a node

B. an antinode

C. a region of minimum amplitude

D. a region of destructive interference

Use the following information to answer the next question.

While a student holds one end of a 6.0 m long spring stationary, another student shakes the other end until four standing waves, or loops, are formed. While maintaining the standing wave pattern, the student shakes the spring 10.0 complete cycles in 6.3 s.

26. Determine the speed of the wave through the spring.

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Use the following information to answer the next question.

The fundamental frequency of a standing wave in a vibrating string is 40.0 Hz.

27. What is the frequency of the third harmonic?

Use the following information to answer the next question.

Two organ pipes are open at both ends and are 25 cm and 28 cm long respectively.

28. If both pipes are producing sound at their fundamental frequencies and the speed of sound is 343 m/s, what is the beat frequency?

Use the following information to answer the next question.

A singer has a vocal tract (from larynx to mouth) that is 15 cm long and the vocal tract behaves like an air column closed at one end by the voice box (larynx).

29. If the singer sings a musical note that predominantly resonates at the fundamental frequency and the air temperature is 28°C, what is the frequency of the note?

30. A student produces a certain note when she blows gently into a clarinet. If she blows harder, the **most likely** change will be that the sound wave will have a

A. larger speed

B. higher frequency

C. greater amplitude

D. longer wavelength

Use the following information to answer the next question.

A man moves away from a stationary horn with a speed equal to 0.5 that of the speed of sound.

31. If the horn is emitting a sound with a frequency *f*, the frequency the man hears is

A. much greater than *f*

B. greater than *f*

C. equal to *f*

D. less than *f*

Use the following information to answer the next question.

The tension on a string is increased by a factor *x*.

32. The velocity of a transverse wave moving through this string will

A. increase by a factor of $x/2$

B. increase by a factor of x

C. increase by a factor of the square of x

D. increase by a factor of the square root of x

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Student Notes and Problems

ANSWERS AND SOLUTIONS

CASTLE ROCK
RESEARCH CORP

INTRODUCTION TO PHYSICS

Lesson 2—Significant Digits and Scientific Notation

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. a) 3

b) 2

c) 3

d) 1

e) 3

3. a) 7.23×10^{1} m

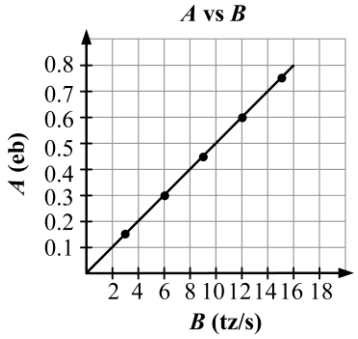
b) 5.59×10^{2} m

c) 4.90×10^{2} m

Lesson 3—Graphing

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. a)



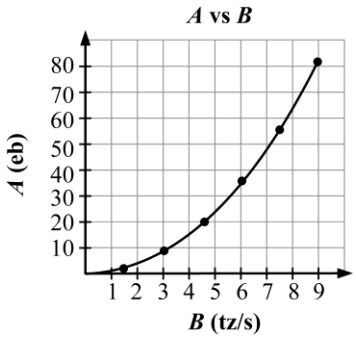
b) $\text{slope} = \dfrac{\text{rise}}{\text{run}} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{0.750 \text{ eb} - 0}{15.0 \text{ tz/s} - 0}$

$= 5.00\times10^{-2} \text{ eb}\cdot\text{s/tz}$

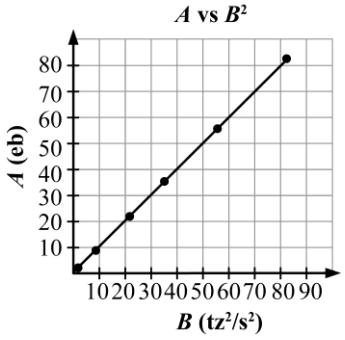
c) $A \propto B$ or $A = kB$

3. a)



b)

B (tz/s)	A (eb)	B^2 (tz^2/s^2)
1.5	2.25	2.25
3.0	9.00	9.00
4.5	20.25	20.25
6.0	36.00	36.00
7.5	56.25	56.25
9.0	81.00	81.00



c) $\text{slope} = \dfrac{\text{rise}}{\text{run}} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{70 \text{ eb} - 0}{70 \text{ tz}^2/\text{s}^2 - 0}$

$= 1.00 \text{ eb}\cdot\text{s}^2/\text{tz}^2$

d) $A \propto B^2$ or $A = \left(1.00 \text{ eb}\cdot\text{s}^2/\text{tz}^2\right)B^2$

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MOTION

Lesson 1—Introduction to Kinematics

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. a) $d = 275\text{ m} + 425\text{ m}$
$= 700\text{ m}$

b) $\vec{d} = 275\text{ m} + (-425\text{ m})$
$= -150\text{ m}$ or 150 m left

3 a) $v_{ave} = \dfrac{d}{t}$
$v_{ave} = \dfrac{(11\text{ m} + 25\text{ m})}{52\text{ m}}$
$= 0.69\text{ m/s}$

b) $\vec{v}_{ave} = \dfrac{\vec{d}}{t}$
$\vec{v}_{ave} = \dfrac{(11\text{ m} + (-25\text{ m}))}{52\text{ s}}$
$= -0.27\text{ m/s}$ or 0.27 m/s left

Lesson 2—Instantaneous Velocity and Speed

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $\vec{v}_{ave} = \dfrac{\vec{d}}{t}$
$= \dfrac{1.00\times10^{2}\text{ m}}{11.2\text{ s}}$
$= 8.93\text{ m/s right}$

3. $v = \dfrac{d}{t}$
$t = \dfrac{d}{v}$
$= \dfrac{2.5\text{ m}}{9.8\text{ m/s}}$
$= 0.26\text{ s}$

5. $\vec{v}_{ave} = \dfrac{\vec{d}}{t}$
$= \dfrac{-64.0\text{ m}}{3.61\text{ s}}$
$= -17.7\text{ m/s}$ or 17.7 m/s down

7. Both speed and velocity are equal to the slope of the graph.
$\text{slope} = \dfrac{\text{rise}}{\text{run}}$
$= \dfrac{(8.0-0)\text{ m}}{(8.0-0)\text{ s}}$
$= 1.0\text{ m/s}$

a) $v = 1.0\text{ m/s}$

b) $\vec{v} = 1.0\text{ m/s right}$

9. Both the displacement and distance are found by determining the area under the graph.
$\text{area} = l \times w$
$= (5.0\text{ s})(2.5\text{ m})$
$= 13\text{ m}$

a) $\vec{d} = 13\text{ m left}$

b) $d = 13\text{ m}$

11. 1st part of the motion:
$\vec{v} = \dfrac{\vec{d}}{t}$
$\vec{d} = \vec{v}t$
$= (8.0\text{ m/s})(25\text{ min} \times 60\text{ s/min})$
$= 1.20\times10^{4}\text{ m right}$

2nd part of the motion:
$\vec{d}_{av} = \vec{v}t$
$= (-5.0\text{ m/s})(15\text{ min} \times 60\text{ s/min})$
$= -4.50\times10^{3}\text{ m}$ or $4.50\times10^{3}\text{ m}$ left

Total displacement:
$\vec{d} = 1.20\times10^{4}\text{ m} - 4.50\times10^{3}\text{ m}$
$= 7.5\times10^{3}\text{ m right}$

Total distance:
$d = 1.20\times10^{4}\text{ m} + 4.50\times10^{3}\text{ m}$
$= 1.65\times10^{4}\text{ m}$

a) $\vec{v}_{ave} = \dfrac{\vec{d}}{t}$

$= \dfrac{7.5\times10^{3}\text{ m}}{40\text{ min}\times60\text{ s/min}}$

$= 3.1\text{ m/s right}$

b) $v_{ave} = \dfrac{d}{t}$

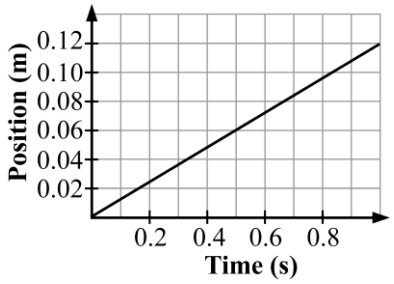
$= \dfrac{1.65\times10^{4}\text{ m}}{40\text{ min}\times60\text{ s/min}}$

$= 6.9\text{ m/s}$

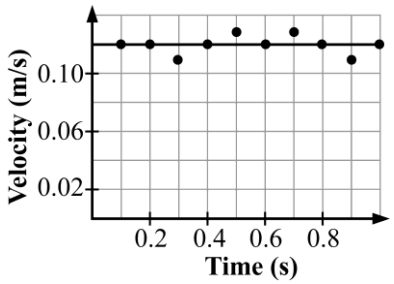
13. a)

time (s)	displacement (m)	displacement during time interval (m)	average velocity during time interval (m/s)
0	0		
		0.012	0.12
0.10	0.012		
		0.012	0.12
0.20	0.024		
		0.011	0.11
0.30	0.035		
		0.012	0.12
0.40	0.047		
		0.013	0.13
0.50	0.060		
		0.012	0.12
0.60	0.072		
		0.013	0.13
0.70	0.085		
		0.012	0.12
0.80	0.097		
		0.011	0.11
0.90	0.108		
		0.012	0.12
1.00	0.120		

b) Position-time graph:



c) Velocity-Time Graph:



d) $\text{slope} = \dfrac{\text{rise}}{\text{run}}$

$= \dfrac{(0.12-0)\text{ m}}{(1.0-0)\text{ s}}$

$\vec{v}_{ave} = 0.12\text{ m/s right}$

e) Displacement is the area under the velocity time graph.

$\text{area} = l\times w$

$= (0.95\text{ s})(0.12\text{ m/s})$

$\vec{d} = 0.11\text{ m right}$

Lesson 3—Uniformly Accelerated Motion in One Direction

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. a) Draw a tangent line at 0.40 s.
Find the slope of this tangent line.

$\text{slope} = \dfrac{\text{rise}}{\text{run}}$

$= \dfrac{(5.0-1.5)\text{ m}}{(0.64-0.30)\text{ s}}$

$\vec{v} = 1.0\times10^{1}\text{ m/s right}$

b) Draw a tangent line at 0.60 s.
Find the slope of this tangent line.

$\text{slope} = \dfrac{\text{rise}}{\text{run}}$

$= \dfrac{(9.0-2.0)\text{ m}}{(0.90-0.38)\text{ s}}$

$\vec{v} = 13\text{ m/s right}$

3. a) Acceleration is the slope of the velocity-time graph.

$\text{slope} = \dfrac{\text{rise}}{\text{run}}$

$= \dfrac{(13.0-0)\text{ m/s}}{(9.0-0)\text{ s}}$

$\vec{a} = 1.4\text{ m/s}^{2}\text{ right}$

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b) Displacement is the area under the velocity-time graph.

$$\text{area} = \frac{1}{2}(l \times w)$$
$$= \frac{1}{2}(10.0\text{ s} \times 14.5\text{ m/s})$$
$$\vec{d} = 73\text{ m right}$$

5. a) Acceleration is the slope of the velocity-time graph.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{(16.0 - 0)\text{ m/s}}{(10.0 - 0)\text{ s}}$$
$$\vec{a} = 1.60\text{ m/s}^2\text{ left}$$

b) Displacement is the area under the velocity-time graph.

$$\text{area} = \frac{1}{2}(l \times w)$$
$$= \frac{1}{2}(10.0\text{ s} \times 16.0\text{ m/s})$$
$$\vec{d} = 80.0\text{ m left}$$

7. a) Acceleration is the slope of the velocity-time graph.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{(32.5 - 10.0)\text{ m/s}}{(10.0 - 0)\text{ s}}$$
$$\vec{a} = 2.25\text{ m/s}^2\text{ right}$$

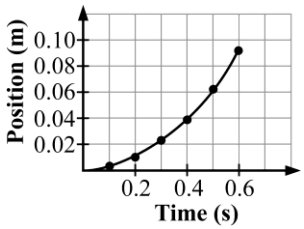
b) Displacement is the area under the velocity-time graph.

$$\text{area} = (l \times w) + \frac{1}{2}(l \times w)$$
$$= (10.0\text{ s} \times 10.0\text{ m/s}) + \frac{1}{2}(10.0\text{ s} \times 22.5\text{ m/s})$$
$$\vec{d} = 213\text{ m right}$$

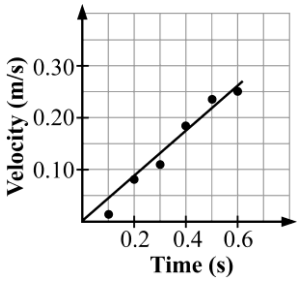
9. a)

Time (s)	Displacement from t = 0 (m)	Displacement during time interval (m)	Average velocity during time interval (m/s)
0	0		
0.10	0.0025	0.0025	0.025
0.20	0.0095	0.007	0.07
0.30	0.022	0.012	0.12
0.40	0.039	0.017	0.17
0.50	0.061	0.022	0.22
0.60	0.086	0.025	0.25

b) Position-Time Graph



c) Velocity-Time Graph



d) Acceleration is the slope of the velocity-time graph.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{(0.22 - 0)\text{ m/s}}{(0.45 - 0)\text{ s}}$$
$$\vec{a} = 0.49\text{ m/s}^2\text{ right}$$

Note: in the following questions, consider right as the positive direction and left as the negative direction.

11.

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
0	–12.0 m/s	?	×	3.40 s

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$
$$= \frac{(-12.0 \text{ m/s}) - 0}{3.40 \text{ s}}$$
$$= -3.53 \text{ m/s}^2 \text{ or } 3.53 \text{ m/s}^2 \text{ left}$$

13.

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
0	?	1.9 m/s^2	?	5.0 s

a) $\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$
$$= \frac{1}{2}(1.9 \text{ m/s}^2)(5.0 \text{ s})^2$$
$$= 24 \text{ m right}$$

b) $\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$
$$1.9 \text{ m/s}^2 = \frac{\vec{v}_f - 0}{5.0 \text{ s}}$$
$$\vec{v}_f = 9.5 \text{ m/s right}$$

c) The magnitude of the displacement (distance) is 24 m.

d) The magnitude of velocity (speed) is 9.5 m/s.

15.

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
5.0 m/s	?	3.0 m/s^2	×	2.9 s

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$
$$3.0 \text{ m/s}^2 = \frac{\vec{v}_f - 5.0 \text{ m/s}}{2.9 \text{ s}}$$
$$\vec{v}_f = (3.0 \text{ m/s}^2)(2.9 \text{ s}) + 5.0 \text{ m/s}$$
$$= 14 \text{ m/s right}$$

17.

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
0	×	?	20.0 m	8.10 s

$$\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$
$$20.0 \text{ m} = \frac{1}{2}\vec{a}(8.10 \text{ s})^2$$
$$\vec{a} = 0.610 \text{ m/s}^2 \text{ right}$$

19.

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
0	122.0 km/h	×	×	10.5 s

$$\vec{v}_{ave} = \frac{\vec{v}_f + \vec{v}_i}{2}$$
$$= \frac{122.0 \text{ km/h} + 0}{2}$$
$$= 61.0 \text{ km/h}$$
$$= 61.0 \text{ km/h} \times 1000 \text{ m/km} \times \frac{1 \text{ h}}{3600 \text{ s}}$$
$$= 16.9 \text{ m/s right}$$

21.

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
0	?	×	19.0 m	7.10 s

$$\vec{d} = \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right)t$$
$$19.0 \text{ m} = \left(\frac{\vec{v}_f + 0}{2}\right)7.10 \text{ s}$$
$$\vec{v}_f = \frac{2(19.0 \text{ m})}{7.10 \text{ s}}$$
$$= 5.35 \text{ m/s right}$$

23.

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
–15.0 m/s	–35.0 m/s	?	–43.0 m	×

$$\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a}\vec{d}$$
$$(-35.0 \text{ m/s})^2 = (-15.0 \text{ m/s})^2 + 2\vec{a}(-43.0 \text{ m})$$
$$\vec{a} = -11.6 \text{ m/s}^2 \text{ or } 11.6 \text{ m/s}^2 \text{ left}$$

25.

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
?	25.0 m/s	1.50 m/s^2	×	10.0 s

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$
$$1.50 \text{ m/s}^2 = \frac{25.0 \text{ m/s} - \vec{v}_i}{10.0 \text{ s}}$$
$$\vec{v}_i = 25.0 \text{ m/s} - (10.0 \text{ s})(1.50 \text{ m/s}^2)$$
$$= 10.0 \text{ m/s right}$$

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27.

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
0	?	×	–31.0 m	5.6 s

$$\vec{d} = \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right)t$$
$$-31.0\text{ m} = \left(\frac{\vec{v}_f + 0}{2}\right)5.6\text{ s}$$
$$\vec{v}_f = \frac{2(-31.0\text{ m})}{5.6\text{ s}}$$
$$= -11\text{ m/s or 11 m/s left}$$

29.

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
0	?	1.4 m/s^2	?	5.0 s

a) $\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$
$$= \frac{1}{2}(1.4\text{ m/s}^2)(5.0\text{ s})^2$$
$$= 18\text{ m down the ramp}$$

b) $\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$
$$1.4\text{ m/s}^2 = \frac{\vec{v}_f - 0}{5.0\text{ s}}$$
$$\vec{v}_f = 7.0\text{ m/s down the ramp}$$

31.

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
0	–12.4 m/s	–3.10 m/s^2	?	4.00 s

You can use any formula except
$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$
$$\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$
$$= \frac{1}{2}(-3.10\text{ m/s}^2)(4.00\text{ s})^2$$
$$= -24.8\text{ m or 24.8 m left}$$

33.

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
0	×	?	19.6 m	?

$$\vec{v}_{ave} = \frac{\vec{d}}{t}$$
$$t = \frac{\vec{d}}{\vec{v}_{ave}}$$
$$= \frac{19.6\text{ m}}{5.00\text{ m/s}}$$
$$= 3.92\text{ s}$$

$$\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$
$$19.6\text{ m} = \frac{1}{2}\vec{a}(3.92\text{ s})^2$$
$$\vec{a} = 2.55\text{ m/s}^2\text{ right}$$

Lesson 4—Freely Falling Objects

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

Note: in all of the following questions, consider up as the positive direction and down as the negative direction.

1.

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
0	?	–9.81 m/s^2	–15.0 m	×

$$\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a}\vec{d}$$
$$= 2(-9.81\text{ m/s}^2)(-15.0\text{ m})$$
$$\vec{v}_f = -17.2\text{ m/s or 17.2 m/s down}$$

3.

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
0	×	–9.81 m/s^2	–1.75 m	?

$$\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$
$$-1.75\text{ m} = \frac{1}{2}(-9.81\text{ m/s}^2)t^2$$
$$t = \sqrt{\frac{2(-1.75\text{ m})}{-9.81\text{ m/s}^2}}$$
$$t = 0.597\text{ s}$$

5.

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
0	?	$-9.81\ \text{m/s}^2$	×	2.5 s

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$
$$-9.81\ \text{m/s}^2 = \frac{\vec{v}_f - 0}{2.5\ \text{s}}$$
$$\vec{v}_f = -25\ \text{m/s or } 25\ \text{m/s down}$$

7.

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
–10.0 m/s	–25.0 m/s	$-9.81\ \text{m/s}^2$	×	?

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$
$$-9.81\ \text{m/s}^2 = \frac{(-25.0\ \text{m/s}) - (-10.0\ \text{m/s})}{t}$$
$$t = \frac{(-25.0\ \text{m/s}) - (-10.0\ \text{m/s})}{-9.81\ \text{m/s}^2}$$
$$= 1.53\ \text{s}$$

9.

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
?	–10.0 m/s	$-9.81\ \text{m/s}^2$	×	0.880 s

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$
$$-9.81\ \text{m/s}^2 = \frac{(-10.0\ \text{m/s}) - \vec{v}_i}{0.880\ \text{s}}$$
$$\vec{v}_i = -1.37\ \text{m/s or } 1.37\ \text{m/s down}$$

11. a)
$$\vec{v}_{ave} = \frac{\vec{v}_f + \vec{v}_i}{2}$$
$$= \frac{50.0\ \text{m/s} + 0}{2}$$
$$= 25.0\ \text{m/s left}$$

b) Read time from the graph.
$t = 4.0$ s

13.
$$\vec{v}_{ave} = \frac{\vec{v}_f + \vec{v}_i}{2}$$
$$-12.0\ \text{m/s} = \frac{\vec{v}_f - 0}{2}$$
$$\vec{v}_f \equiv -24.0\ \text{m/s or } 24.0\ \text{m/s down}$$

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
0	–24.0 m/s	$-9.81\ \text{m/s}^2$	×	?

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$
$$t = \frac{(-24.0\ \text{m/s}) - 0}{-9.81\ \text{m/s}^2}$$
$$= 2.45\ \text{s}$$

15.

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
0	–19.6 m/s	?	–24.0 m	×

$$\vec{v}_f^{\,2} = \vec{v}_i^{\,2} + 2\vec{a}\vec{d}$$
$$(-19.6\ \text{m/s})^2 = 2\vec{a}(-24.0\ \text{m})$$
$$\vec{a} = \frac{(-19.6\ \text{m/s})^2}{2(-24.0\ \text{m})}$$
$$= -8.00\ \text{m/s}^2 \text{ or } 8.00\ \text{m/s}^2 \text{ down}$$

17.

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
×	–15.0 m/s	$-9.81\ \text{m/s}^2$	–10.0 m	?

Find initial velocity first:
$$\vec{v}_f^{\,2} = \vec{v}_i^{\,2} + 2\vec{a}\vec{d}$$
$$(-15.0\ \text{m/s})^2 = \vec{v}_i^{\,2} + 2(-9.81\ \text{m/s}^2)(-10.0\ \text{m})$$
$$\vec{v}_i = -5.367\ \text{m/s or } 5.367\ \text{m/s down}$$

Now use any equation that you like to find time:
$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$
$$-9.81\ \text{m/s}^2 = \frac{(-15.0\ \text{m/s}) - (-5.367\ \text{m/s})}{t}$$
$$t = 0.982\ \text{s}$$

19.

$\vec{v}_i$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
0	?	?	40.0 m	×

$$\vec{v}_{ave} = \frac{\vec{v}_i + \vec{v}_f}{2}$$
$$-30.0\ \text{m/s} = \frac{0 + \vec{v}_f}{2}$$
$$\vec{v}_f = -60.0\ \text{m/s or } 60.0\ \text{m/s down}$$

$$\vec{v}_f^{\,2} = \vec{v}_i^{\,2} + 2\vec{a}\vec{d}$$
$$(-60.0\ \text{m/s})^2 = 2\vec{a}(-40.0\ \text{m})$$
$$\vec{a} = -45.0\ \text{m/s or } 45.0\ \text{m/s down}$$

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Lesson 5—Horizontal Uniformly Accelerated Motion in Two Directions along a Straight Line

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. Velocity is the slope of position-time graph. To find velocity, draw a tangent line at the point you are investigating and find the slope of the tangent line.

a) $\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{(14 - 5.0)\text{ m}}{(6.2 - 0)\text{ s}}$

$\vec{v} = 1.5$ m/s up

b) $\text{slope} = \frac{(5.0 - 14)\text{ m}}{(10.0 - 3.8)\text{ s}}$

$\vec{v} = -1.5$ m/s or 1.5 m/s down

3. a) Displacement is read from graph.

$\vec{d} = 13.0$ m right

b) Velocity is the slope of the position-time graph.

$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{(7.0 - 0)\text{ m}}{(7.0 - 0)\text{ s}}$

$\vec{v} = 1.0$ m/s right

c) Velocity is the slope of the position-time graph.

slope = 0

$\vec{v} = 0$

d) Velocity is the slope of the position-time graph.

$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{(13.0 - 7.0)\text{ m}}{(16.0 - 13.0)\text{ s}}$

$\vec{v} = 2.0$ m/s right

e) $\vec{v}_{av} = \frac{\vec{d}}{t}$

$= \frac{13.0\text{ m}}{16.0\text{ s}}$

$= 0.81$ m/s right

f) Since slope does not change, the velocity does not change.

$\vec{a} = 0$

5. a) Velocity is the slope of the position-time graph.

$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{(-16.0 - 16.0)\text{ m}}{(8.0 - 0)\text{ s}}$

$\vec{v} = -4.0$ m/s or 4.0 m/s left

b) the slope is the same as part a).

$\vec{v} = -4.0$ m/s or 4.0 m/s left

c) Velocity does not change.

$\vec{a} = 0$

7. a) Displacement is the area under the velocity-time graph.

greatest area = A

b) greatest speed = B

9. a) D

b) A

c) C

d) B, D

11. a) Read the velocity from the velocity-time graph.

greatest velocity = A

b) Acceleration is the slope of the velocity-time graph.

slope = zero during A

c) Acceleration is the slope of the velocity-time graph.

steepest slope = D

Lesson 6—Vertical Uniformly Accelerated Motion in Two Directions along a Straight Line

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1.

$\vec{v}_0$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
14.0 m/s	×	–9.81 m/s^2	?	1.80 s

$$\vec{d} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$
$$= (14.0 \text{ m/s})(1.80 \text{ s}) + \frac{1}{2}(-9.81 \text{ m/s}^2)(1.80 \text{ s})^2$$
$$= 9.31 \text{ m upward}$$

3.

$\vec{v}_0$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
11.0 m/s	–7.3 m/s	?	×	9.3 s

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_0}{t}$$
$$= \frac{(-7.3 \text{ m/s}) - 11.0 \text{ m/s}}{9.3 \text{ s}}$$
$$= -2.0 \text{ m/s}^2 \text{ or } 2.0 \text{ m/s}^2 \text{ down the slope}$$

5.

$\vec{v}_0$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
15.0 m/s	–8.0 m/s	–9.81 m/s^2	?	×

$$\vec{v}_f^2 = \vec{v}_0^2 + 2\vec{a}\vec{d}$$
$$(-8.0 \text{ m/s})^2 = (15.0 \text{ m/s})^2 + 2(-9.81 \text{m/s}^2)\vec{d}$$
$$\vec{d} = \frac{(-8.0 \text{ m/s})^2 - (15.0 \text{ m/s})^2}{2(-9.81 \text{ m/s}^2)}$$
$$= 8.2 \text{ m up}$$

7.

$\vec{v}_0$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
?	0	×	2.6 m	3.6 s

$$\vec{d} = \left(\frac{\vec{v}_f + \vec{v}_0}{2}\right)t$$
$$2.6 \text{ m} = \left(\frac{0 + \vec{v}_0}{2}\right)(3.6 \text{ s})$$
$$\vec{v}_0 = \frac{2(2.6 \text{ m})}{3.6 \text{ s}}$$
$$= 1.4 \text{ m/s up}$$

9.

$\vec{v}_0$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
25.0 m/s	?	–9.81 m/s^2	×	3.0 s

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_0}{t}$$
$$-9.81 \text{ m/s}^2 = \frac{\vec{v}_f - 25.0 \text{ m/s}}{3.0 \text{ s}}$$
$$\vec{v}_f = (3.0 \text{ s})(-9.81 \text{ m/s}^2) + 25.0 \text{ m/s}$$
$$\vec{v}_f = -4.4 \text{ m/s or } 4.4 \text{ m/s down}$$

11.

$\vec{v}_0$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
?	×	–9.81 m/s^2	5.0 m	3.0 s

$$\vec{d} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$
$$5.0 \text{ m} = \vec{v}_0 (3.0 \text{ s}) + \frac{1}{2}(-9.81 \text{ m/s}^2)(3.0 \text{ s})^2$$
$$\vec{v}_0 = \frac{5.0 \text{ m} - \frac{1}{2}(-9.81 \text{ m/s}^2)(3.0 \text{ s})^2}{3.0 \text{ s}}$$
$$= 16 \text{ m/s up}$$

13.

$\vec{v}_0$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
2.5 m/s	–1.6 m/s	?	1.0 m	×

$$\vec{v}_f^2 = \vec{v}_0^2 + 2\vec{a}\vec{d}$$
$$(-1.6 \text{ m/s})^2 = (2.5 \text{ m/s})^2 + 2\vec{a}(1.0 \text{ m})$$
$$\vec{a} = \frac{(-1.6 \text{ m/s})^2 - (2.5 \text{ m/s})^2}{2(1.0 \text{ m})}$$
$$= -1.8 \text{ m/s}^2$$
$$\text{or } 1.8 \text{ m/s}^2 \text{ down the slope}$$

15.

$\vec{v}_0$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
5.0 m/s	–5.0 m/s	?	0	3.0 s

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_0}{t}$$
$$= \frac{(-5.0 \text{ m/s}) - 5.0 \text{ m/s}}{3.0 \text{ s}}$$
$$= -3.3 \text{ m/s}^2 \text{ or } 3.3 \text{ m/s}^2 \text{ down}$$

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17.

$\vec{v}_0$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
5.0 m/s	×	?	0	3.0 s

$$\vec{d} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$
$$0 = (5.0 \text{ m/s})(3.0 \text{ s}) + \frac{1}{2}\vec{a}(3.0 \text{ s})^2$$
$$\vec{a} = -\frac{2(5.0 \text{ m})}{(3.0 \text{ s})}$$
$$\vec{a} = -3.3 \text{ m/s}^2 \text{ or } 3.3 \text{ m/s}^2 \text{ down}$$

19.

$\vec{v}_0$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
×	0	-9.81 m/s^2	?	2.65 s

If it takes 5.30 s to go up and down, it will take $\frac{5.30 \text{ s}}{2} = 2.65$ s to reach the highest point.

Now find the final velocity:

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_0}{t}$$
$$-9.81 \text{ m/s}^2 = \frac{0 - \vec{v}_0}{2.65 \text{ s}}$$
$$\vec{v}_0 = 26.0 \text{ m/s}$$

Find the displacement:

$$\vec{d} = \left(\frac{\vec{v}_f + \vec{v}_0}{2}\right)t$$
$$= \left(\frac{0 + 26.0 \text{ m/s}}{2}\right)(2.65 \text{ s})$$
$$= 34.4 \text{ m}$$

21.

$\vec{v}_0$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
14.0 m/s	0	-9.81 m/s^2	?	?

a) $\vec{v}_f^2 = \vec{v}_0^2 + 2\vec{a}\vec{d}$
$$0 = (14.0 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)\vec{d}$$
$$\vec{d} = \frac{-(14.0 \text{ m/s})^2}{2(-9.81 \text{ m/s}^2)}$$
$$= 9.99 \text{ m up}$$

b)
$$\vec{a} = \frac{\vec{v}_f - \vec{v}_0}{t}$$
$$-9.81 \text{ m/s}^2 = \frac{0 - 14.0 \text{ m/s}}{t}$$
$$t = 1.43 \text{ s}$$

Lesson 7—Projectile Motion

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. Find the horizontal component of displacement.

Find t from vertical component:

$\vec{v}_{y0}$	$\vec{v}_{yf}$	$\vec{a}_y$	$\vec{d}_y$	t
0	×	-9.81 m/s^2	-90.0 m	?

$$\vec{d}_y = \vec{v}_{y0} t + \frac{1}{2}\vec{a}_y t^2$$
$$-90.0 \text{ m} = \frac{1}{2}(-9.81 \text{ m/s}^2)t^2$$
$$t = \sqrt{\frac{2(-90.0 \text{ m})}{-9.81 \text{ m/s}^2}}$$
$$= 4.28 \text{ s}$$

Now use t to find the horizontal component of displacement:

$$\vec{v}_x = \frac{\vec{d}_x}{t}$$
$$\vec{d}_x = \vec{v}_x t$$
$$= (10.0 \text{ m/s})(4.28 \text{ s})$$
$$= 42.8 \text{ m}$$

3. Find the vertical component of displacement.

Find t from the horizontal component:

$$\vec{v}_x = \frac{\vec{d}_x}{t}$$
$$t = \frac{\vec{d}_x}{\vec{v}_x}$$
$$= \frac{100.0 \text{ m}}{18.0 \text{ m/s}}$$
$$= 5.56 \text{ s}$$

Now use t to find the vertical component of displacement:

$\vec{v}_{y0}$	$\vec{v}_{yf}$	$\vec{a}_y$	$\vec{d}_y$	t
0	×	-9.81 m/s^2	?	5.56 s

$$\vec{d}_y = \vec{v}_{y0} t + \frac{1}{2}\vec{a}_y t^2$$
$$= \frac{1}{2}(-9.81 \text{ m/s}^2)(5.56 \text{ s})^2$$
$$= -152 \text{ m}$$

$\therefore$ the height of the cliff is 152 m.

5. Find the vertical component of displacement.

$\vec{v}_{y0}$	$\vec{v}_{yf}$	$\vec{a}_y$	$\vec{d}_y$	t
0	×	$-9.81\ \text{m/s}^2$	?	5.50 s

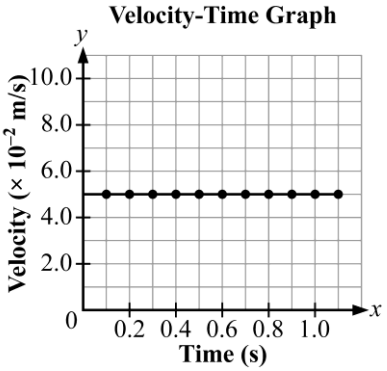
$$\vec{d}_y = \vec{v}_{y0}t + \frac{1}{2}\vec{a}_y t^2$$
$$= \frac{1}{2}\left(-9.81\ \text{m/s}^2\right)\left(5.50\ \text{s}\right)^2$$
$$= -148\ \text{m}$$

$\therefore$ the height of the building is 148 m.

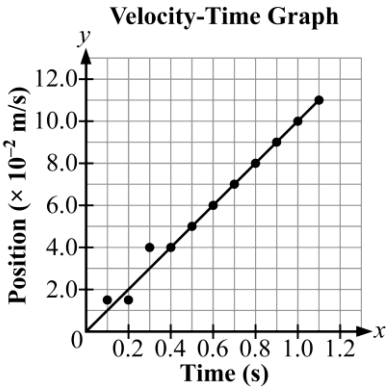
7. **a)**

Time (s)	**Displacement from t = 0**	**Displacement during time interval (× 10 ^{3} m)**		**Average velocity during time interval (× 10^2 m/s)**	
0	0	**horiz.**	**vert.**	**horiz.**	**vert.**
0.10	0.5	5.0	1.5	5.0	1.5
0.20	1.0	5.0	1.5	5.0	1.5
0.30	1.5	5.0	4.0	5.0	4.0
0.40	2.0	5.0	4.0	5.0	4.0
0.50	2.5	5.0	5.0	5.0	5.0
0.60	3.0	5.0	6.0	5.0	6.0
0.70	3.5	5.0	7.0	5.0	7.0
0.80	4.0	5.0	8.0	5.0	8.0
0.90	4.5	5.0	9.0	5.0	9.0
1.00	5.0	5.0	10.0	5.0	10.0
1.10	5.5	5.0	11.0	5.0	11.0

b)



c)



d) **i)** slope = 0
$\therefore$ acceleration = 0

ii) $\text{slope} = \dfrac{\text{rise}}{\text{run}}$
$$= \frac{(0.10 - 0.01)\ \text{m/s}}{(0.95 - 0.05)\ \text{s}}$$
$\vec{a}_y = 0.10\ \text{m/s}^2$ or $0.10\ \text{m/s}^2$ down

Practice Test

ANSWERS AND SOLUTIONS

1. The motion of a falling object is uniform accelerated motion. This means that the acceleration of a falling object remains constant. Remember, the slope of a velocity-time graph is the acceleration. All of A., B., and D., represent a constant slope; but A. shows no change in velocity; D. shows that the velocity is decreasing.

B is the answer.

3. The acceleration of a falling object is constant. Therefore you are looking for a graph that shows a constant acceleration.

A is the answer.

5. Velocity is slope of a position-time graph

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{50\ \text{m} - 20\ \text{m}}{6\ \text{s}}$$
$$\bar{v} = 5.0\ \text{m/s right}$$

A is the answer.

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7. The magnitude of the velocity will be at a maximum at the height it was thrown and zero at maximum height. The direction of the vector will be up as it moves toward its maximum height. It will have no direction at the moment the velocity is zero. The direction of the velocity vector will be down as the ball falls back towards the player.

9. The slope of a position-time graph is the velocity. In this graph, the slope is decreasing. Therefore, the velocity is decreasing.

11. The magnitude of the displacement vector will increase as it moves up the ramp. As the ball rolls back down, it will decrease but remain positive until reaching the point that it was released where it will equal zero. As it rolls further, the displacement will become negative and continue to decrease as it rolls down. The direction will always point up as the ball reaches its maximum height and then falls back down to the point of release. After falling below the point of the release, the displacement vector will always point downward.

13.

$\vec{v}_0$	$\vec{v}_f$	$\vec{a}$	$\vec{d}$	t
15 m/s	×	–9.81 m/s^2	?	8.00 s

$$\vec{d} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$
$$= (15 \text{ m/s})(8.00 \text{ s}) + \frac{1}{2}(-9.81 \text{ m/s}^2)(8.00)^2$$
$$= -1.9 \times 10^2 \text{ m or } 1.9 \times 10^2 \text{ m down}$$

The height is the magnitude of the displacement.
$\therefore d = 1.9 \times 10^2$ m

B is the answer.

15. a) The objects hit the ground at the same velocity.

b) The object thrown upward will reach a velocity of zero due to the downward acceleration of gravity at some point above the student. It will then fall until it reaches the height that it was thrown. At this point, its velocity will be 5.0 m/s down because the acceleration is constant. This is the same velocity that the second object is thrown downward. Since both objects have the same velocity at the same height, they will hit the ground with the same velocity.

17. All objects fall at the same rate if you ignore air friction. Also, the vertical component of the motion is independent of the horizontal component.

C is the answer.

19. All objects fall at the same rate. Also the vertical component of the motion is independent of the horizontal component. Therefore, the lower the cliff, the sooner the object will hit the ground.

C is the answer.

21. They all have zero acceleration. Slope of a position-time graph represents the velocity. All these graphs have a constant slope. Therefore, they have a constant velocity. If the velocity is constant, there is no acceleration.

23. Sections A, B, C have a constant slope, which means that in these sections they have a constant velocity. At any instant in time, the steepest slope can be found at the end of section D.

D is the answer.

25. The object falls to the ground due to a change in velocity as it increases downward.
The acceleration of the object is constant due to gravity.

FORCES

Lesson 1—Forces and Newton's Laws of Motion

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1.
$$\vec{F}_{net} = m\vec{a}$$
$$\vec{a} = \frac{\vec{F}_{net}}{m}$$
$$= \frac{9.0 \text{ N}}{20.0 \text{ kg}}$$
$$= 0.45 \text{ m/s}^2 \text{ right}$$

3. $\vec{F}_{net} = m\vec{a}$
$= (16.0 \text{ kg})(-2.0 \text{ m/s}^2)$
$= -32 \text{ N or } 32 \text{ N left}$

5. $\vec{F}_{net} = m\vec{a}$
$= (5.2 \text{ kg})(6.0 \text{ m/s}^2)$
$= 31 \text{ N right}$

7. $\vec{a} = \dfrac{\vec{v}_f - \vec{v}_0}{t}$
$= \dfrac{(-25.0 \text{ m/s}) - 0}{10.0 \text{ s}}$
$= -2.500 \text{ m/s}^2 \text{ or } 2.500 \text{ m/s}^2 \text{ left}$

$\vec{F}_{net} = m\vec{a}$
$= (925 \text{ kg})(-2.500 \text{ m/s}^2)$
$= -2.31 \times 10^3 \text{ N or } 2.31 \times 10^3 \text{ N left}$

9. $\vec{v}_f^2 = \vec{v}_0^2 + 2\vec{a}\vec{d}$
$(12 \text{ m/s})^2 = (5.0 \text{ m/s})^2 + 2\vec{a}(94 \text{ m})$
$\vec{a} = 0.633 \text{m/s}^2 \text{ right}$

$\vec{F}_{net} = m\vec{a}$
$= (1.20 \times 10^3 \text{ kg})(0.633 \text{ m/s}^2)$
$= 7.6 \times 10^2 \text{ N right}$

11. a) $\vec{F}_{net} = m\vec{a}$
$\vec{a} = \dfrac{\vec{F}_{net}}{m}$
$= \dfrac{6.6 \text{ N}}{9.0 \text{ kg}}$
$= 0.733 \text{ m/s}^2 \text{ right}$

$\vec{v}_f^2 = \vec{v}_0^2 + 2\vec{a}\vec{d}$
$(3.0 \text{ m/s})^2 = 2(0.733 \text{ m/s}^2)\vec{d}$
$\vec{d} = 6.1 \text{ m right}$

b) $\vec{a} = \dfrac{\vec{v}_f - \vec{v}_0}{t}$
$0.733 \text{ m/s}^2 = \dfrac{3.0 \text{ m/s} - 0}{t}$
$t = 4.1 \text{ s}$

Lesson 2—Newton's Third Law of Motion

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $\vec{F}_1 = -\vec{F}_2$
$m_A\vec{a}_A = -m_B\vec{a}_B$
$(38.0 \text{ kg})(0.60 \text{ m/s}^2) = -m_B(-0.75 \text{ m/s}^2)$
$m_B = 3.0 \times 10^1 \text{ kg}$

3. $\vec{a} = \dfrac{\vec{v}_f - \vec{v}_0}{t}$
$= \dfrac{22 \text{ m/s} - 11 \text{ m/s}}{0.75 \text{ s}}$
$= 14.7 \text{ m/s}^2 \text{ right}$

$\vec{F} = m\vec{a}$
$= (9.8 \times 10^3 \text{ kg})(14.7 \text{ m/s}^2)$
$= 1.4 \times 10^5 \text{ N right}$

Lesson 3—Force Due to Gravity

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $\vec{F}_g = m\vec{g}$
$= (25.0 \text{ kg})(-9.81 \text{ m/s}^2)$
$= -245 \text{ N or } 245 \text{ N down}$

3. $\vec{F}_g = m\vec{g}$
$\vec{g} = \dfrac{\vec{F}_g}{m}$
$= \dfrac{-36.0 \text{ N}}{22.0 \text{ kg}}$
$= -1.64 \text{ m/s}^2 \text{ or } 1.64 \text{ m/s}^2 \text{ down}$

5. $\vec{F}_g = m\vec{g}$
$m = \dfrac{\vec{F}_g}{\vec{g}}$
$= \dfrac{-127 \text{ N}}{-9.81 \text{ m/s}^2}$
$= 12.9 \text{ kg}$

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Lesson 4—Contact Forces

PRACTICE EXERCISES
ANSWERS AND SOLUTIONS

1. $\vec{F}_N = -m\vec{g}$
$= -(14.0 \text{ kg})(-9.81 \text{ m/s}^2)$
$= 137 \text{ N up}$

3. $F_f = \mu_k F_N$
$\mu_k = \dfrac{F_f}{F_N}$
$= \dfrac{3.0 \text{ N}}{20.0 \text{ N}}$
$= 0.15$

5. $\vec{F}_{net} = m\vec{a}$
$= (6.2 \text{ kg})(1.1 \text{ m/s}^2)$
$= 6.8 \text{ N right}$

The net force is the sum of all forces acting on an object. This can be written as:
$\vec{F}_{net} = \vec{T} + \vec{F}_f$
$\vec{F}_f = \vec{F}_{net} - \vec{T}$
$= 6.8 \text{ N} - 22.0 \text{ N}$
$= -15.2 \text{ N or } 15.2 \text{ N left}$

The force of friction is negative as expected since it is in the opposite direction of motion.

The coefficient of friction can be found using:
$F_f = \mu F_N$

Remember that for this equation only the magnitude of the force of friction and normal force are used. First, find the normal force:
$\vec{F}_N = -m\vec{g}$
$\vec{F}_N = -(6.2 \text{ kg})(-9.81 \text{ m/s}^2)$
$\vec{F}_N = 60.8 \text{ N up}$

$F_f = \mu_k F_N$
$\mu_k = \dfrac{F_f}{F_N}$
$= \dfrac{15.2 \text{ N}}{60.8 \text{ N}}$
$= 0.25$

7. Since the direction of the spring force is left, choose it as negative.
$\vec{F}_s = -k\vec{x}$
$\vec{x} = -\dfrac{\vec{F}_s}{k}$
$= -\dfrac{(-2.0 \text{ N})}{15 \text{ N/m}}$
$= 0.13 \text{ m right}$

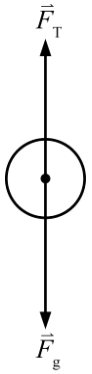
9. Choose the direction of displacement as negative because it is down. This makes the value of the spring force positive because it must be in the opposite direction.
$\vec{F}_s = -k\vec{x}$
$k = -\dfrac{\vec{F}_s}{\vec{x}}$
$= -\dfrac{1.65 \text{ N}}{(-0.110 \text{ m})}$
$= 15 \text{ N/m}$

11. $\vec{F}_s = -k\vec{x}$
$\vec{x} = -\dfrac{\vec{F}_s}{k}$
$= -\dfrac{9.3 \text{ N}}{25 \text{ N/m}}$
$= -0.37 \text{ m or } 0.37 \text{ m down}$

Lesson 6—Forces and Motion

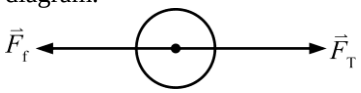
PRACTICE EXERCISES
ANSWERS AND SOLUTIONS

1. Consider up to be the positive direction.



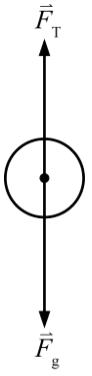
$\sum \vec{F} = \vec{F}_{net}$
$= \vec{F}_T + \vec{F}_g$
$= 145 \text{ N} + m\vec{g}$
$= 145 \text{ N} + (11.0 \text{ kg})(-9.81 \text{ m/s}^2)$
$= 37.09 \text{ N}$
$\vec{F}_{net} = m\vec{a}$
$a = \dfrac{F_{net}}{m}$
$= \dfrac{37.09 \text{ N}}{11.0 \text{ kg}}$
$= 3.37 \text{ m/s}^2$

3. Consider right to be the positive direction given the diagram.



$$\begin{aligned}\sum \vec{F} &= \vec{F}_{net}\\ &= m\vec{a}\\ &= (20.0\text{ kg})(0.80\text{ m/s}^2)\\ &= 16.0\text{ N}\\ \vec{F}_{net} &= \vec{F}_T + \vec{F}_f\\ \vec{F}_f &= \vec{F}_{net} - \vec{F}_T\\ &= 16\text{ N} - 27.0\text{ N}\\ &= -11\text{ N}\\ F_f &= 11\text{ N}\end{aligned}$$

5. **a)** Consider up as the positive direction.



$$\begin{aligned}\sum \vec{F} &= \vec{F}_{net}\\ &= m\vec{a}\\ &= (1.20\times10^3\text{ kg})(-1.05\text{ m/s}^2)\\ &= -1.260\times10^3\text{ N}\\ \vec{F}_g &= m\vec{g}\\ &= (1.20\times10^3\text{ kg})(-9.81\text{ m/s}^2)\\ &= -1.177\times10^4\text{ N}\\ \vec{F}_{net} &= \vec{F}_g + \vec{F}_T\\ \vec{F}_T &= \vec{F}_{net} - \vec{F}_g\\ &= -1.260\times10^3\text{ N} - (-1.177\times10^4\text{ N})\\ &= 1.05\times10^4\text{ N or } 1.05\times10^4\text{ N up}\end{aligned}$$

b) Consider up as the positive direction.

$$\begin{aligned}\sum \vec{F} &= \vec{F}_{net}\\ &= m\vec{a}\\ &= (1.20\times10^3\text{ kg})(1.05\text{ m/s}^2)\\ &= 1.260\times10^3\text{ N}\\ \vec{F}_{net} &= \vec{F}_T + \vec{F}_g\\ \vec{F}_T &= \vec{F}_{net} - \vec{F}_g\\ &= (1.260\times10^3\text{ N}) - (-1.177\times10^4\text{ N})\\ &= 1.30\times10^4\text{ N}\end{aligned}$$

c) acceleration $= 0$; $\therefore\ \sum \vec{F} = 0$

Consider up as the positive direction.

$$\begin{aligned}\sum \vec{F} &= \vec{F}_{net}\\ &= \vec{F}_T + \vec{F}_g\\ &= 0\\ &\text{or}\\ \vec{F}_T &= -\vec{F}_g\\ &= -m\vec{g}\\ &= -(1.20\times10^3\text{ kg})(-9.81\text{ m/s}^2)\\ &= 1.18\times10^4\text{ N}\end{aligned}$$

7. Consider west as the positive direction.

$$\begin{aligned}\vec{v}_f{}^2 &= \vec{v}_i{}^2 + 2\vec{a}\vec{d}\\ \vec{a} &= \frac{(\vec{v}_f{}^2 - \vec{v}_i{}^2)}{2\vec{d}}\\ &= \frac{(0 - (0.50\text{ m/s})^2)}{2(0.25\text{ m})}\\ &= -0.50\text{ m/s}^2\\ \vec{F}_{net} &= \vec{F}_f\\ &= m\vec{a}\\ &= (1.0\text{ kg})(-0.50\text{ m/s}^2)\\ &= -0.50\text{ N}\\ \therefore F_f &= 0.50\text{ N}\end{aligned}$$

9. Consider north as the positive direction.

$$\begin{aligned}\vec{F} &= m\vec{a}\\ &= (7.00\text{ kg})(2.30\text{ m/s}^2\text{ north})\\ &= 16.1\text{ N north}\end{aligned}$$

The magnitude of the horizontal force required to accelerate the given mass at 2.30 m/s^2 is 16.1 N.

11. Consider east to be the positive direction.

$$\begin{aligned}\sum \vec{F} &= \vec{F}_{net} = m\vec{a}\\ \vec{a} &= \frac{\vec{F}_{net}}{m}\\ &= \frac{-2.50\times10^4\text{ N}}{1.20\times10^3\text{ kg}}\\ &= -20.83\text{ m/s}^2\end{aligned}$$

$$\begin{aligned}\vec{a} &= \frac{\vec{v}_f - \vec{v}_i}{t}\\ -20.83\text{ m/s}^2 &= \frac{\vec{v}_f - 20.0\text{ m/s}}{0.500\text{ s}}\\ \vec{v}_f &= (-20.83\text{ m/s}^2)(0.500\text{ s}) + 20.0\text{ m/s}\\ \vec{v}_f &= 9.58\text{ m/s or } 9.58\text{ m/s east}\end{aligned}$$

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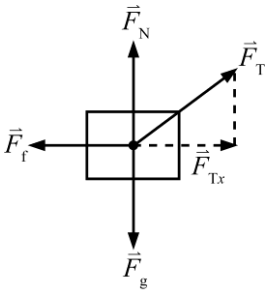
13.



Consider right and up as the positive directions.

$$\cos\theta = \frac{F_{Tx}}{F_T}$$
$$F_{Tx} = F_T \cos\theta$$
$$= (60.0\text{ N})(\cos 42.0°)$$
$$= 44.59\text{ N}$$
$$\vec{F}_{Tx} = 44.59\text{ N or }44.59\text{ N to the right}$$
$$\vec{F}_g = m\vec{g}$$
$$m = \frac{\vec{F}_g}{\vec{g}}$$
$$= \frac{-125\text{ N}}{-9.81\text{ m/s}^2}$$
$$= 12.74\text{ kg}$$
$$\sum \vec{F} = \vec{F}_{net} = \vec{F}_{Tx} + \vec{F}_f$$
$$= 44.59\text{ N} + (-15.0\text{ N})$$
$$= 29.59\text{ N}$$
$$\vec{F}_{net} = m\vec{a}$$
$$\vec{a} = \frac{\vec{F}_{net}}{m}$$
$$= \frac{29.59\text{ N}}{12.74\text{ kg}}$$
$$= 2.32\text{ m/s}^2 \text{ or } 2.32\text{ m/s}^2 \text{ to the right}$$

15. Consider north as the positive direction.

$$\vec{v}_f{}^2 = \vec{v}_i{}^2 + 2\vec{a}\vec{d}$$
$$0 = (3.0\text{ m/s})^2 + 2(\vec{a})(8.0\text{ m})$$
$$\vec{a} = -\frac{(3.0\text{ m/s})^2}{2(8.0\text{ m})}$$
$$\vec{a} = -0.563\text{ m/s}^2$$
$$\vec{F}_f = m\vec{a}$$
$$= (0.48\text{ kg})(-0.563\text{ m/s}^2)$$
$$= -0.27\text{ N or }0.27\text{ N south}$$

Practice Test

ANSWERS AND SOLUTIONS

1. $F_f = \mu F_N$

$$\mu = \frac{F_f}{F_N}$$

The coefficient of friction is the ratio of the force of friction to the normal force between two surfaces in contact. The higher or lower the coefficient, the higher or lower the frictional force that exists due to the contact of the two surfaces.

3. The graph is linear, and this implies a direct relationship between the variables. This confirms Newton's second law, which states that the acceleration varies directly with the net force. Mathematically, you have $a \propto F_{net}$.

$$\text{units of slope} = \frac{\text{rise units}}{\text{run units}}$$
$$= \frac{\text{m/s}^2}{\text{N}}$$
$$= \frac{\text{m/s}^2}{\text{kg}\cdot\text{m/s}^2}$$
$$= \frac{1}{\text{kg}} = \text{kg}^{-1}$$

The slope is the reciprocal of the mass. That is, the mass of the object can be found by calculating the slope of this graph and determining its inverse.

5. Consider up as the positive direction.

$$\vec{F}_g = m\vec{g}$$
$$m = \frac{\vec{F}_g}{\vec{g}}$$
$$= \frac{-50.0\text{ N}}{-9.81\text{ m/s}^2}$$
$$= 5.10\text{ kg}$$

7. The gravitational force will be toward each other. The particular direction depends on which object the force is with respect to.

9. The force of friction will remain constant at 8.0 N with a direction opposing motion until the object comes to a complete stop. Once stopped, the magnitude of the force of friction becomes zero and it has no direction.

11. On a horizontal surface:

$\vec{F}_N = -\vec{F}_g$

Because the three objects have the same masses, they all experience the same magnitude of normal force.

13. a) Newton's third law applies because boy A exerts force of 250 N on boy B, and boy B exerts a force of 250 N on boy A. These two forces are equal in magnitude but opposite in direction.

These two forces cannot be added in a sensible way because they act on different objects. In fact, this is the defining attribute of an action-reaction couple.

b) Boy A exerts a force of 250 N on the rope, and boy B exerts a force of 250 N on the rope in the opposite direction. The tension in the rope is 250 N throughout.

The vector sum of the forces is zero. The forces can now be added because both forces act on the rope.

$\Sigma\vec{F} = 250\text{ N} + (-250\text{ N})$
$= 0\text{ N}$

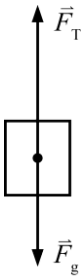
There would be no acceleration of either of the boys since the net force transferred to the through the rope is zero.

15. Choosing right as the positive direction:

$\vec{F}_{net} = m\vec{a}$
$= (2.0\text{ kg})(-1.6\text{ m/s}^2)$
$= -3.2\text{ N or } 3.2\text{ N left}$

$\vec{F}_{net} = \vec{T}_1 + \vec{T}_2$
$\vec{T}_2 = \vec{F}_{net} - \vec{T}_1$
$\vec{T}_2 = -3.2\text{ N} - (-17\text{ N})$
$\vec{T}_2 = 14\text{ N right}$
$T_2 = 14\text{ N}$

17. The net force can be determined from the free-body diagram. The tension in the cord is transferred around the pulley and acts on the mass in the upward direction. Consider up as the positive direction.



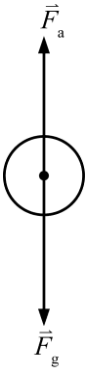
$\vec{F}_{net} = \vec{F}_T + \vec{F}_g$
$= \vec{F}_T + m\vec{g}$
$= 85.0\text{ N}$
$+ (5.00\text{ kg})(-9.81\text{ m/s}^2)$
$= 35.95\text{ N or } 35.95\text{ N up}$

Now, the acceleration of the system can be determined, and from that, the time can be calculated.

$\vec{a} = \dfrac{\vec{F}_{net}}{m}$
$= \dfrac{35.95\text{ N}}{5.0\text{ kg}}$
$= 7.19\text{ m/s}^2 \text{ or } 7.19\text{ m/s}^2$

$\vec{d} = \vec{v}_i t + \dfrac{1}{2}\vec{a}t^2$
$\vec{d} = 0 + \dfrac{1}{2}\vec{a}t^2$
$t = \sqrt{\dfrac{2\vec{d}}{\vec{a}}}$
$t = \sqrt{\dfrac{2(1.00\text{ m})}{7.19\text{ m/s}^2}}$
$= 0.527\text{ s}$

19.



$\vec{F}_{net} = m\vec{a}$
$= (1.50\text{ kg})(1.20\text{ m/s}^2)$
$= 1.80\text{ N up}$
$\vec{F}_g = m\vec{g}$
$= (1.50\text{ kg})(-9.80\text{ m/s}^2)$
$= -14.7\text{ N or } 14.7\text{ N down}$
$\vec{F}_{net} = \vec{F}_a + \vec{F}_g$
$\vec{F}_a = \vec{F}_{net} - \vec{F}_g$
$= 1.80\text{ N} - (-14.7\text{ N})$
$= 16.5\text{ N up}$

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ENERGY

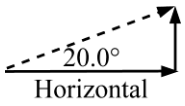
Lesson 1—Work

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $W = Fd$
$= (20.0\text{ N})(1.50\text{ m})$
$= 30.0\text{ J}$

3. $W = Fd$
$= (2.20\text{ N})(0)$
$= 0$

5. Find the horizontal component of the force.



$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$\text{adjacent} = (\cos 20.0°)(75.0\text{ N})$
$= 70.48\text{ N}$
$W = Fd$
$= (70.48\text{ N})(10.0\text{ m})$
$= 705\text{ J}$

7. $W = Fd$
$= (0)(9.0\text{ m})$
$= 0$

9. Work done against friction:
$W = Fd$
$= (3.8\text{ N})(6.0\text{ m})$
$= 22.8\text{ J}$

Work done to accelerate object:
$$\vec{d} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$
$$6.0\text{ m} = \frac{1}{2}\vec{a}(4.0\text{ s})^2$$
$$\vec{a} = 0.750\text{ m/s}^2$$

$\vec{F} = m\vec{a}$
$= (25.0\text{ kg})(0.750\text{ m/s}^2)$
$= 18.8\text{ N}$

$W = Fd$
$= (18.8\text{ N})(6.0\text{ m})$
$= 1.13\times10^2\text{ J}$

The total work done is:
$W = 1.13\times10^2\text{ J} + 22.8\text{ J}$
$= 1.4\times10^2\text{ J}$

11. Work is the area under a force-displacement graph.
$W = Fd$
$= (3.5\text{ N})(16.0\text{ m})$
$= 56\text{ J}$

Lesson 2—Energy

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $E_p = Fd$
$= (25.0\text{ N})(2.10\text{ m})$
$= 52.5\text{ J}$

3. $E_p = mgh$
$= (2.75\text{ kg})(9.81\text{ m/s}^2)(7.00\text{ m})$
$= 189\text{ J}$

5. $E_p = mgh$
$= (2.00\times10^2\text{ kg})(9.81\text{ m/s}^2)(6.0\text{ m})$
$= 1.2\times10^4\text{ J}$

Lesson 3—Kinetic Energy

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $E_k = \frac{1}{2}mv^2$
$= \frac{1}{2}(3.0\text{ kg})(7.5\text{ m/s})^2$
$= 84\text{ J}$

3. $\vec{v}_f^2 = \vec{v}_0^2 + 2\vec{a}\vec{d}$

$= 2\left(2.5 \text{ m/s}^2\right)\left(15.0 \text{ m}\right)$

$\vec{v}_f = 8.66 \text{ m/s}$

$\vec{F}_g = m\vec{g}$

$m = \dfrac{\vec{F}_g}{\vec{g}}$

$= \dfrac{-10.0 \text{ N}}{-9.81 \text{ m/s}^2}$

$= 1.02 \text{ kg}$

$E_k = \dfrac{1}{2}mv^2$

$= \dfrac{1}{2}\left(1.02 \text{ kg}\right)\left(8.66 \text{ m/s}\right)^2$

$= 38 \text{ J}$

5. $\vec{F}_g = m\vec{g}$

$m = \dfrac{\vec{F}_g}{\vec{g}}$

$= \dfrac{-10.0 \text{ N}}{-9.8! \text{ m/s}^2}$

$= 1.02 \text{ kg}$

$E_k = \dfrac{1}{2}mv^2$

$v = \sqrt{\dfrac{2E_k}{m}}$

$= \sqrt{\dfrac{2\left(3.00 \times 10^2 \text{ J}\right)}{1.02 \text{ kg}}}$

$= 24.3 \text{ m/s}$

Lesson 4—Law of Conservation of Energy

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $\Delta E_k + \Delta E_p = 0$

$\Delta E_k = -\Delta E_p$

$\dfrac{1}{2}m\left(v_f^2 - v_0^2\right) = -mg\Delta h$

$\dfrac{1}{2}\left(v_f^2 - v_0^2\right) = -g\left(h_f - h_0\right)$

$\dfrac{1}{2}\left(\left(3.2 \text{ m/s}\right)^2 - 0\right) = -\left(9.81 \text{ m/s}^2\right)\left(0 - h_0\right)$

$h_0 = \dfrac{\frac{1}{2}\left(3.2 \text{ m/s}\right)^2}{\left(9.81 \text{ m/s}^2\right)}$

$= 0.52 \text{ m}$

3. $\Delta E_k + \Delta E_p = 0$

$\Delta E_k = -\Delta E_p$

$\dfrac{1}{2}m\left(v_f^2 - v_0^2\right) = -mg\Delta h$

$\dfrac{1}{2}\left(v_f^2 - v_0^2\right) = -g\left(h_f - h_0\right)$

$\dfrac{1}{2}\left(\left(37.0 \text{ m/s}\right)^2 - 0\right) = -\left(9.81 \text{ m/s}^2\right)\left(0 - h_0\right)$

$h_0 = \dfrac{\frac{1}{2}\left(37.0 \text{ m/s}\right)^2}{9.81 \text{ m/s}^2}$

$= 69.8 \text{ m}$

5. $\Delta E_k + \Delta E_p = 0$

$\Delta E_k = -\Delta E_p$

$\dfrac{1}{2}m\left(v_f^2 - v_0^2\right) = -mg\Delta h$

$\dfrac{1}{2}\left(v_f^2 - v_0^2\right) = -g\Delta h$

$\dfrac{1}{2}\left(v_f^2 - 0\right) = -\left(9.81 \text{ m/s}^2\right)\left(-4.0 \text{ m}\right)$

$v_f = \sqrt{-\left(9.81 \text{ m/s}^2\right)\left(-4.0 \text{ m}\right)\left(2\right)}$

$= 8.9 \text{ m/s}$

7. Find h_0 first:

$\sin 30.0° = \dfrac{h_0}{12.0 \text{ m}}$

$h_0 = 6.0 \text{ m}$

$\Delta E_k + \Delta E_p = 0$

$\Delta E_k = -\Delta E_p$

$\dfrac{1}{2}m\left(v_f^2 - v_0^2\right) = -mg\Delta h$

$\dfrac{1}{2}\left(v_f^2 - v_0^2\right) = -g\Delta h$

$\dfrac{1}{2}\left(v_f^2 - 0\right) = -\left(9.81 \text{ m/s}^2\right)\left(-6.0 \text{ m}\right)$

$v_f = \sqrt{-\left(9.81 \text{ m/s}^2\right)\left(-6.0 \text{ m}\right)\left(2\right)}$

$= 10.8 \text{ m/s}$

9. $\Delta E_k + \Delta E_p = 0$

$\Delta E_k = -\Delta E_p$

$\dfrac{1}{2}m\left(v_f^2 - v_0^2\right) = -mg\Delta h$

$\dfrac{1}{2}\left(v_f^2 - v_0^2\right) = -g\Delta h$

$\dfrac{1}{2}\left(v_f^2 - 0\right) = -\left(9.81 \text{ m/s}^2\right)\left(-10.0 \text{ m}\right)$

$v_f = \sqrt{-\left(9.81 \text{ m/s}^2\right)\left(-10.0 \text{ m}\right)\left(2\right)}$

$= 14 \text{ m/s}$

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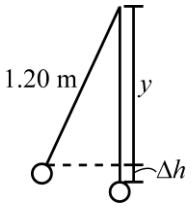
11.



Find Δh first:

$$\cos 25° = \frac{y}{1.2\text{ m}}$$
$$y = 1.09\text{ m}$$
$$\therefore \Delta h = 1.2\text{ m} - y$$
$$= 1.2\text{ m} - 1.09\text{ m}$$
$$= 0.11\text{ m}$$
$$\Delta E_k + \Delta E_p = 0$$
$$\Delta E_k = -\Delta E_p$$
$$\frac{1}{2}m\left(v_f^2 - v_0^2\right) = -mg\Delta h$$
$$\frac{1}{2}\left(v_f^2 - v_0^2\right) = -g\Delta h$$
$$\frac{1}{2}\left(v_f^2 - 0\right) = -\left(9.81\text{ m/s}^2\right)\left(-0.11\text{ m}\right)$$
$$v_f = \sqrt{-\left(9.81\text{ m/s}^2\right)\left(-0.11\text{ m}\right)(2)}$$
$$= 1.47\text{ m/s}$$

13.
$$\Delta E_k + \Delta E_p = 0$$
$$\Delta E_k = -\Delta E_p$$
$$\frac{1}{2}m\left(v_f^2 - v_0^2\right) = -mg\Delta h$$
$$\frac{1}{2}\left(v_f^2 - v_0^2\right) = -g\Delta h$$
$$\frac{1}{2}\left(v_f^2 - 0\right) = -\left(9.81\text{ m/s}^2\right)\left(-2.0\text{ m}\right)$$
$$v_f = \sqrt{-\left(9.81\text{ m/s}^2\right)\left(-2.0\text{ m}\right)(2)}$$
$$= 6.3\text{ m/s}$$

Lesson 5—Power

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $P = \frac{W}{t}$ and $W = mgh$
$$t = \frac{mgh}{P}$$
$$= \frac{\left(45.0\text{ kg}\right)\left(9.81\text{ m/s}^2\right)\left(6.0\text{ m}\right)}{1.50\times10^3\text{ W}}$$
$$= 1.8\text{ s}$$

3.
$$\vec{v}_{av} = \frac{\vec{v}_f + \vec{v}_0}{2}$$
$$= \frac{3.00\text{ m/s} + 0}{2}$$
$$= 1.50\text{ m/s}$$

$$\vec{v}_f^2 = \vec{v}_0^2 + 2\vec{a}\vec{d}$$
$$\left(3.00\text{ m/s}\right)^2 = 2\vec{a}\left(1.5\text{ m}\right)$$
$$\vec{a} = 3.0\text{ m/s}^2$$

$$\vec{F} = m\vec{a}$$
$$= \left(2.00\text{ kg}\right)\left(3.0\text{ m/s}^2\right)$$
$$= 6.0\text{ N}$$

$$P = Fv_{av}$$
$$= \left(6.0\text{ N}\right)\left(1.50\text{ m/s}\right)$$
$$= 9.0\text{ W}$$

5.
$$\vec{v}_f^2 = \vec{v}_0^2 + 2\vec{a}\vec{d}$$
$$\left(6.0\text{ m/s}\right)^2 = 2\vec{a}\left(2.0\text{ m}\right)$$
$$\vec{a} = +9.00\text{ m/s}^2$$

$$\vec{F}_{net} = m\vec{a}$$
$$= \left(5.0\text{ kg}\right)\left(9.00\text{ m/s}^2\right)$$
$$= +45.0\text{ N}$$

$$\vec{F}_{net} = \vec{F}_a + \vec{F}_f$$
$$\vec{F}_a = \vec{F}_{net} - \vec{F}_f$$
$$= 45.0\text{ N} - \left(-4.0\text{ N}\right)$$
$$= +49.0\text{ N}$$

$$\vec{v}_{av} = \frac{\vec{v}_f + \vec{v}_0}{2}$$
$$= \frac{6.0\text{ m/s} + 0}{2}$$
$$= +3.00\text{ m/s}$$

$$P = F_a v$$
$$= \left(49.0\text{ N}\right)\left(3.00\text{ m/s}\right)$$
$$= 1.5\times10^2\text{ W}$$

7. $\text{efficiency} = \dfrac{\text{power out}}{\text{power in}} \times 100\%$

$$\text{power out} = \frac{\text{efficiency} \times \text{power in}}{100\%}$$
$$= \frac{(82\%)(1.00 \times 10^5 \text{ W})}{100\%}$$
$$= \frac{(0.82)(1.00 \times 10^5 \text{ W})}{1.00}$$
$$= 8.2 \times 10^4 \text{ W}$$

$$\text{power out} = \frac{W}{t} \text{ and } W = mgh$$
$$t = \frac{mgh}{\text{power out}}$$
$$= \frac{(50.0 \text{ kg})(9.81 \text{ m/s}^2)(8.00 \text{ m})}{8.2 \times 10^4 \text{ W}}$$
$$= 0.0479 \text{ s}$$

Lesson 6—Machines and Efficiency

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $\text{work out} = mgh$
$$= (225 \text{ kg})(9.81 \text{ m/s}^2)(1.20 \text{ m})$$
$$= 2649 \text{ J}$$
$\text{work in} = Fd$
$$= (315 \text{ N})(10.0 \text{ m})$$
$$= 3150 \text{ J}$$

$$\text{efficiency} = \frac{\text{work out}}{\text{work in}} \times 100\%$$
$$= \frac{2649 \text{ J}}{3150 \text{ J}} \times 100\%$$
$$= 84.1\%$$

3. $\text{work out} = mgh$
$$= (75.0 \text{ kg})(9.81 \text{ m/s}^2)(3.0 \text{ m})$$
$$= 2207 \text{ J}$$

$$\text{efficiency} = \frac{\text{work out}}{\text{work in}} \times 100\%$$
$$\text{work in} = \frac{2207 \text{ J}}{78.5\%} \times 100\%$$
$$= \frac{2205 \text{ J}}{0.785} \times 1.00$$
$$= 2809 \text{ J}$$

$\text{work in} = Fd$
$$F = \frac{\text{work in}}{d}$$
$$= \frac{2809 \text{ J}}{8.0 \text{ m}}$$
$$= 3.5 \times 10^2 \text{ N}$$

5. $\text{work out} = mgh$
$$= (65.0 \text{ kg})(9.81 \text{ m/s}^2)(1.92 \text{ m})$$
$$= 1224 \text{ J}$$

$$\text{efficiency} = \frac{\text{work out}}{\text{work in}} \times 100\%$$
$$\text{work in} = \frac{1224}{68.2\%} \times 100\%$$
$$= \frac{1223 \text{ J}}{0.682} \times 1.00$$
$$= 1795 \text{ J}$$

Work required to overcome friction:
$\text{W} = \text{work in} - \text{work out}$
$$= 1795 \text{ J} - 1224 \text{ J}$$
$$= 571 \text{ J}$$
$W = F_f d$
$$= \frac{571 \text{ J}}{5.00 \text{ m}}$$
$$= 114 \text{ N}$$

Lesson 7—Electric Current

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $I = \dfrac{q}{t}$
$q = It$
$$= (3.60\text{A})(15.3 \text{ s})$$
$$= 55.1 \text{ C}$$

$$\text{e}^- = \frac{q}{1.60 \times 10^{-19} \text{ C}}$$
$$= \frac{55.1 \text{ C}}{1.60 \times 10^{-19} \text{ C}}$$
$$= 3.44 \times 10^{20}$$

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3. $I = \dfrac{q}{t}$

$= \dfrac{5.60\text{ C}}{15.4\text{ s}}$

$= 3.64\times10^{-1}\text{ A}$

5. $P = I^2R$

$= (11.0\text{ A})^2\,(7.20\text{ W})$

$= 8.71\times10^2\text{ W}$

$P = \dfrac{\Delta E}{t}$

$\Delta E = Pt$

$= (8.71\times10^2\text{ W})(25.0\text{ s})$

$= 2.18\times10^4\text{ J}$

7. $P = IV$

$I = \dfrac{P}{V}$

$= \dfrac{4.00\times10^2\text{ W}}{1.20\times10^2\text{ V}}$

$= 3.33\text{ A}$

9. **a)** $V = IR$

$= (20.0\text{A})(6.00\ \Omega)$

$= 1.20\times10^2\text{ V}$

b) $q = It$

$= (20.0\text{ A})(60.0\text{ s})$

$= 1200\text{ C}$

$\text{e}^- = \dfrac{\text{total charge}}{\text{charge on 1 e}^-}$

$= \dfrac{1\ 200\text{ C}}{1.60\times10^{-19}\text{ C}}$

$= 7.50\times10^{21}$

11. $P = \dfrac{\Delta E}{t}$

$\Delta E = Pt$

$= (1.0\text{ kW})(5.0\text{ h})$

$= 5.0\text{ kW}\cdot\text{h}$

$\text{cost} = (\$0.060)(5.0\text{ kW}\cdot\text{h})$

$= \$0.30$

13. $W = mgh$

$= (45\text{ kg})(9.81\text{ m/s}^2)(9.0\text{ m})$

$= 3973\text{ J}$

25% of $7.5\times10^2\text{ W} = 187.5\text{ W}$

$P = \dfrac{W}{t}$

$t = \dfrac{W}{P}$

$= \dfrac{3\ 973\text{ J}}{187.5\text{ W}}$

$= 21\text{ s}$

Lesson 8—Electric Circuits

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. In a series circuit:

$I = I_1 = I_2 = I_3$

$I_1 = 1.7\text{ A}$

$I_2 = 1.7\text{ A}$

$I_3 = 1.7\text{ A}$

3. In a series circuit:

$V = V_1 + V_2$

$12.0\text{ V} = 8.0\text{ V} + V_2$

$V_2 = 4.0\text{ V}$

5. At first consider the series circuit:

$V = V_1 + V_2$

$45.0\text{ V} = 11.0\text{ V} + V_2$

$V_2 = 45.0\text{ V} - 11.0\text{ V}$

$= 34.0\text{ V}$

Then consider the parallel circuit:

$V_2 = V_3$

$34.0\text{ V} = V_2 = V_3$

$\therefore V_3 = 34.0\text{ V}$

7. In a parallel circuit:

$\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

$= \dfrac{1}{6.0\ \Omega} + \dfrac{1}{8.0\ \Omega}$

$R_{eq} = 3.4\ \Omega$

9. In a series circuit:

$$\begin{aligned} R_{eq} &= R_1 + R_2 + R_3 \\ &= 9.0\ \Omega + 3.0\ \Omega + 12.0\ \Omega \\ &= 24.0\ \Omega \end{aligned}$$

11. In a parallel circuit:

$$\begin{aligned} \frac{1}{R_{eq}} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \\ \frac{1}{2.0\ \Omega} &= \frac{1}{4.5\ \Omega} + \frac{1}{9.0\ \Omega} + \frac{1}{R_3} \\ \frac{1}{R_3} &= \frac{1}{2.0\ \Omega} - \left(\frac{1}{4.5\ \Omega} + \frac{1}{9.0\ \Omega} \right) \\ R_3 &= 6.0\ \Omega \end{aligned}$$

13. a) In a series circuit:

$$\begin{aligned} R_{eq} &= R_1 + R_2 \\ &= 10.0\ \Omega + 15.0\ \Omega \\ &= 25.0\ \Omega \\ I &= \frac{V}{R_{eq}} \\ &= \frac{30.0\ \text{V}}{25.0\ \Omega} \\ &= 1.20\ \text{A} \end{aligned}$$

$$I = I_1 = I_2$$

$$\therefore\ I_1 = 1.20\ \text{A}$$

$$I_2 = 1.20\ \text{A}$$

b)

$$\begin{aligned} P &= I^2 R_1 \\ &= (1.20\ \text{A})^2\ (10.0\ \Omega) \\ &= 14.4\ \text{W} \end{aligned}$$

15. a) In a parallel circuit:

$$\begin{aligned} \frac{1}{R_{eq}} &= \frac{1}{R_1} + \frac{1}{R_2} \\ &= \frac{1}{6.0\ \Omega} + \frac{1}{8.0\ \Omega} \\ R_{eq} &= 3.43\ \Omega \end{aligned}$$

In the second circuit:

$$\begin{aligned} I_2 &= \frac{V_2}{R_2} \\ &= \frac{12.0\ \text{V}}{8.0\ \Omega} \\ &= 1.5\ \text{A} \\ I_1 &= \frac{V}{R_{eq}} \\ &= \frac{12.0\ \text{V}}{3.43\ \Omega} \\ &= 3.5\ \text{A} \end{aligned}$$

b)

$$\begin{aligned} P &= I^2 R \\ &= (3.5\ \text{A})^2\ (3.43\ \Omega) \\ &= 42\ \text{W} \end{aligned}$$

17. a) In a series circuit:

$$\begin{aligned} R_{eq} &= R_1 + R_2 + R_3 \\ &= 2.0\ \Omega + 2.5\ \Omega + 3.0\ \Omega \\ &= 7.5\ \Omega \\ V &= IR \\ &= IR_{eq} \\ &= (8.0\ \text{A})(7.5\ \Omega) \\ &= 6.0 \times 10^1\ \text{V} \end{aligned}$$

b)

$$\begin{aligned} P &= I^2 R \\ &= (8.0\ \text{A})^2\ (7.5\ \text{W}) \\ &= 4.8 \times 10^2\ \text{W} \end{aligned}$$

19.

$$I_1 = \frac{V}{R_{eq(total)}}$$

$$\begin{aligned} R_{eq(total)} &= \frac{V}{I_{eq}} \\ &= \frac{90.0\ \text{V}}{4.5\ \text{A}} \\ &= 20.0\ \Omega \end{aligned}$$

Add R_1 and R_2 for parallel circuit I:

$$\begin{aligned} \frac{1}{R^{\text{I}}_{eq}} &= \frac{1}{20.0\ \Omega} + \frac{1}{10.0\ \Omega} \\ R^{\text{I}}_{eq} &= 6.67\ \Omega \end{aligned}$$

Add R_4 and R_5 for parallel circuit II:

$$\begin{aligned} \frac{1}{R^{\text{II}}_{eq}} &= \frac{1}{20.0\ \Omega} + \frac{1}{10.0\ \Omega} \\ R^{\text{II}}_{eq} &= 6.67\ \Omega \end{aligned}$$

Find R_3 :

In a series circuit:

$$\begin{aligned} R_{eq(total)} &= R^{\text{I}}_{eq} + R^{\text{II}}_{eq} + R_3 \\ R_3 &= R_{eq(total)} - \left(R^{\text{I}}_{eq} + R^{\text{II}}_{eq} \right) \\ &= 20.0\ \Omega - (6.67\ \Omega + 6.67\ \Omega) \\ &= 6.7\ \Omega \end{aligned}$$

Now, find the current:

$$I_3 = 4.5\ \text{A}$$

The sum of the current through R_1 and R_2 is also 4.5 A.

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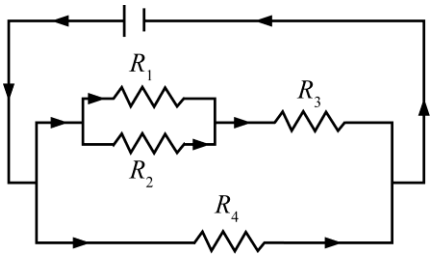
Since $I \alpha \frac{1}{R}$, the current through R_1 will be 0.50 the current through R_2.

Let $I_{R_2} = I_2$

$$I_{T(\text{through} R_1 \text{ and } R_2)} = I_{R_1} + I_{R_2}$$
$$4.5 \text{ A} = 0.50\, I_2 + I_2$$
$$4.5 \text{ A} = 1.50\, I_2$$
$$I_2 = 1.5 \text{ A}$$

In this problem:
$I_4 = I_2 = 1.5 \text{ A}$

21. Find total resistance of the circuit:



Add R_1 and R_2

$$\frac{1}{R_{eq(1)}} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$= \frac{1}{12\ \Omega} + \frac{1}{12\ \Omega}$$
$$R_{eq(1)} = 6.0\ \Omega$$

Add 6.0 Ω to R_3

$$R_{eq(2)} = 6.0 \text{ W} + 12 \text{ W}$$
$$= 18 \text{ W}$$

Add 18 Ω to R_4

$$\frac{1}{R_{eq(3)}} = \frac{1}{12\ \Omega} + \frac{1}{18\ \Omega}$$
$$R_{eq(3)} = 7.2\ \Omega$$

Find power dissipated by the circuit:

$$P = \frac{V^2}{R}$$
$$= \frac{(120 \text{ V})^2}{7.2\ \Omega}$$
$$= 2.0 \times 10^3 \text{ W}$$

Find energy used in 4.0 h

$$P = \frac{\Delta E}{t}$$
$$\Delta E = Pt$$
$$= \left(2.0 \times 10^4 \text{ W}\right)(4.0 \text{ h} \times 3600 \text{ s/h})$$
$$= 2.9 \times 10^7 \text{ J}$$

Use: $\Delta E = mc\Delta T$

$$\Delta T = \frac{\Delta E}{mC}$$
$$= \frac{2.9 \times 10^7 \text{ J}}{(200 \text{ kg})(4.19 \times 10^3 \text{ J/kg} \cdot \text{C})}$$
$$= 34° \text{ C}$$

Final temperature:

$$T_f = T_i + \Delta T$$
$$= 15°\text{C} + 34°\text{C}$$
$$= 49°\text{C}$$

Lesson 9—Electromotive Force (EMF)

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $V = \mathcal{E} - Ir$
$= 1.5 \text{ V} - (1.0 \text{ A})(0.50\ \Omega)$
$= 1.5 \text{ V} - 0.50 \text{ V}$
$= 1.0 \text{ V}$

3. Current flowing to battery
$V = \mathcal{E} + Ir$
$= 24 \text{ V} + (24 \text{ A})(0.25\ \Omega)$
$= 30 \text{ V}$

5. $V = \mathcal{E} - Ir$

$$r = \frac{\mathcal{E} - V}{I}$$
$$= \frac{120 \text{ V} - 115 \text{ V}}{12 \text{ A}}$$
$$= 0.42\ \Omega$$

Lesson 10—Thermal Energy

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $\Delta E_h = m\Delta T c$
$= (0.462\,\text{kg})(80°\text{C} - 24°\text{C})\left(4.18\times10^3\ \text{J}/(\text{kg}\cdot°\text{C})\right)$
$= 1.08\times10^5\ \text{J}$

3. $\Delta E_h = m\Delta T c$
$\Delta T = \dfrac{\Delta E_h}{mc}$
$= \dfrac{2.50\times10^4\ \text{J}}{(0.200\ \text{kg})\left(3.47\times10^3\ \text{J}/(\text{kg}\cdot°\text{C})\right)}$
$= 36.0°\text{C}$
$\Delta T = T_f - T_i$
$T_f = T_i + \Delta T$
$= 20.0°\text{C} + 36.0°\text{C}$
$= 56.0°\text{C}$

5. heat gained by cold water + heat lost by hot water = 0
$\Delta E_{hC} + \Delta E_{hH} = 0$
$\Delta E_{hC} = -\Delta E_{hH}$
$m_C\Delta T_C c_C = -m_H\Delta T_H c_H$
$(0.185\,\text{kg})(T_f - 12.0°\text{C})\left(4.18\times10^3\,\text{J}/(\text{kg}\cdot°\text{C})\right)$
$= -(0.295\,\text{kg})(T_f - 85.0°\text{C})\left(4.18\times10^3\,\text{J}/(\text{kg}\cdot°\text{C})\right)$
$\left(7.733\times10^2\ \text{J}/°\text{C}\right)(T_f - 12.0°\text{C})$
$= -\left(1.233\times10^3\,\text{J}/°\text{C}\right)(T_f - 85.0°\text{C})$
$\left(7.733\times10^2\ \text{J}/°\text{C}\right)T_f - 9.280\times10^3\ \text{J}$
$= -\left(1.233\times10^3\,\text{J}/°\text{C}\right)T_f + 1.048\times10^5\ \text{J}$
$\left(7.733\times10^2\ \text{J}/°\text{C}\right)T_f + \left(1.233\times10^3\,\text{J}/°\text{C}\right)T_f$
$= 1.048\times10^5\ \text{J} + 9.280\times10^3\ \text{J}$
$\left(2.006\times10^3\ \text{J}/°\text{C}\right)T_f = 1.14\times10^5\ \text{J}$
$T_f = \dfrac{1.140\times10^5\ \text{J}}{2.006\times10^3\ \text{J}/°\text{C}}$
$= 56.8°\text{C}$

7. heat gained by water + heat lost by metal = 0
$\Delta E_{hW} + \Delta E_{hM} = 0$
$\Delta E_{hW} = -\Delta E_{hM}$
$m_W\Delta T_W c_W = -m_M\Delta T_M c_M$
$(0.265\,\text{kg})(33.0°\text{C} - 26.0°\text{C})\left(4.18\times10^3\,\text{J}/(\text{kg}\cdot°\text{C})\right)$
$= -(0.352\ \text{kg})(33.0°\text{C} - 215°\text{C})c_M$
$7.754\times10^3\ \text{J} = (64.06\ \text{kg}\cdot°\text{C})c_M$
$c_M = 1.21\times10^2\ \text{J}/(\text{kg}\cdot°\text{C})$

9. heat gained by cold liquid + heat lost by warm liquid = 0
$\Delta E_{hW} + \Delta E_{hC} = 0$
$\Delta E_{hW} = -\Delta E_{hC}$
$m_W\Delta T_W c_W = -m_C\Delta T_C c_C$, where $(c_C = c_W)$
$\Delta E_{hC} + \Delta E_{hW} = 0$
$m_C\Delta T_C c_C + m_W\Delta T_W c_W = 0$, where $(c_C = c_W)$
$(29.0\ \text{g})(T_f - 52.0°\text{C})c = -(20.0\ \text{g})(T_f - 10.0°\text{C})c$
Divide both sides by *c*.
$(29.0\ \text{g})(T_f - 52.0°\text{C}) = -(20.0\ \text{g})(T_f - 10.0°\text{C})$
$(29.0)T_f - 1.508\times10^3\ \text{g}\cdot°\text{C}$
$= -(20.0\ \text{g})T_f + 200.0\ \text{g}\cdot°\text{C}$
$(29.0)T_f + (20.0\ \text{g})T_f$
$= 200.0\ \text{g}\cdot°\text{C} + 1.508\times10^3\ \text{g}\cdot°\text{C}$
$(49.0\ \text{g})T_f = 1.708\times10^3\ \text{g}\cdot°\text{C}$
$T_f = \dfrac{1.708\times10^3\ \text{g}\cdot°\text{C}}{49.0\ \text{g}}$
$= 34.9°\text{C}$
Note: You do not need to change the mass to kilograms because the mass unit cancels out in the end.

Practice Test

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. When work is done in lifting an object, the gravitational potential energy is increased. When work is done on an object in accelerating it, the kinetic energy is changed. When work is done on an object to overcome friction, mechanical energy is changed to thermal energy.

3. $W = Fd$
No work is done because there is no displacement.

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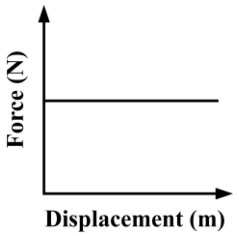
5. If the velocity is constant, there is no net force. Therefore, the applied force is equal to the force of friction. Note that the force of friction is constant. This means that the applied force to overcome friction is also constant. The following graph shows a sketch of what this scenario should look like.



7. Substitute $\frac{1}{2}v$ into the equation for kinetic energy to determine the effect of halving the velocity of an object in motion.

$$E_k = \frac{1}{2}mv^2$$

$$E_{k\,1/2} = \frac{1}{2}m\left(\frac{1}{2}v\right)^2$$

$$= \frac{1}{2}m\left(\frac{1}{4}v^2\right)$$

$$= \frac{1}{4}\left(\frac{1}{2}mv^2\right)$$

$$= \frac{1}{4}E_k$$

$\therefore$ When v is halved, E_k is quartered.

9. Gravity is a conservative force. Spring forces are conservative forces. These are conservative because the amount of work done is independent of the path taken. Mechanical energy is conserved.

11. $P = Fv$

$= mgv$

$= (45\text{ kg})(9.81\text{ m/s}^2)(0.25\text{ m/s})$

$= 110\text{ W}$

Lifting a 45 kg student vertically at a speed of 0.25 m/s will require 110 W.

13. $\Delta E_h = m\Delta Tc$

The temperature change in metal bar 2 is greater than the change in metal bar 1. This means that metal bar 2 has a smaller specific heat capacity then metal bar 1. It takes less energy to raise the substance's temperature, but because the same amount of heat was applied to each bar, metal bar 2 reaches a higher temperature.

15. $\Delta E_{hX} + \Delta E_{hY} = 0$

$$m_X\Delta T_X c_X = -m_Y\Delta T_Y c_Y$$

$$(1\text{ kg})(34°\text{C} - 10°\text{C})(c_X) = -(2\text{ kg})(34°\text{C} - 58°\text{C})(c_Y)$$

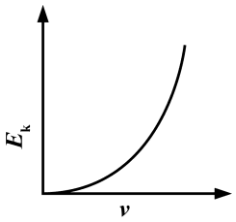
$$24c_X = 48c_Y$$

$$c_X = 2.0c_Y$$

Therefore, liquid X has the higher specific heat capacity. Its specific heat capacity is 2.0 times greater than the specific heat capacity of liquid Y.

17. $E_k = \frac{1}{2}mv^2$ or $E_k \propto v^2$

Therefore, as the velocity increases, the kinetic energy will also increase but at an increasing rate. The following sketch illustrates this exponential growth in graph form.



19. $V = IR$

$= (9.0\text{ A})(5.0\ \Omega)$

$= 45\text{ V}$

The potential difference across a 5.0 Ω resistor is 45 V.

21. $P = \frac{\Delta E}{t}$

$\Delta E = Pt$

$= (775\text{ W})(60\text{ s/min})(15\text{ min})$

$= 7.0 \times 10^5\text{ J}$

A 775 W heating coil will use 7.0×10^5 J of energy when it works for 15 minutes.

23. $R_{eq} = R_1 + R_2$
$= 5.00\ \Omega + 7.50\ \Omega$
$= 12.5\ \Omega$

The equivalent resistance of the two resistors is 12.5 Ω.

25. For every electron that passes through R_2, two electrons will pass through R_3 (R_3 has $\frac{1}{2}$ the resistance). This means that for every three electrons that pass through this circuit, one electron passes through R_2 (i.e., $\frac{1}{3}$ of the total current passes through R_2).

$I_{R_2} = \frac{1}{3} \times I_{R_2}$
$= \frac{1}{3} \times 9.0\ \text{A}$
$= 3.0\ \text{A}$

The current through R_2 is 3.0 A.

27. $V = IR \ \therefore\ R = \frac{V}{I}$

If two copper conductors have the same resistance, they must have the same voltage-to-current ratio.

29. By adding R_2 in parallel, the resistance in the circuit decreases, but the resistance of R_1 does not change. The voltage across each resistor in a parallel circuit also does not change by adding R_2.
$P = \frac{V^2}{R}$

Therefore, the power dissipated by R_1 also does not change when resistor R_2 is added in parallel.

WAVES AND SOUND

Lesson 1—Waves and Sound

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $v = \lambda f$
$\lambda = \frac{v}{f}$
$= \frac{3.3 \times 10^{-1}\ \text{m/s}}{5.0 \times 10^{-1}\ \text{Hz}}$
$= 0.66\ \text{m}$

3. $v = \lambda f$
$f = \frac{2.5\ \text{m/s}}{5.0\ \text{m}}$
$= 0.500\ \text{Hz}$
$T = \frac{1}{f}$
$= \frac{1}{0.500\ \text{Hz}}$
$= 2.0\ \text{s}$

5. $f = \frac{1}{T}$
$= \frac{1}{1.00 \times 10^{-2}\ \text{Hz}}$
$= 1.00 \times 10^{2}\ \text{Hz}$
$v = \lambda f$
$\lambda = \frac{v}{f}$
$= \frac{335\ \text{m/s}}{1.00 \times 10^{2}\ \text{Hz}}$
$= 3.35\ \text{m}$

7. $v = \lambda f$
$f = \frac{v}{\lambda}$
$= \frac{3.00 \times 10^{8}\ \text{m/s}}{5.00 \times 10^{-7}\ \text{m}}$
$= 6.000 \times 10^{14}\ \text{Hz}$
$T = \frac{1}{f}$
$= \frac{1}{6.000 \times 10^{14}\ \text{Hz}}$
$= 1.67 \times 10^{-15}\ \text{s}$

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Lesson 2—Properties of Waves

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. 2.3 cm + 1.9 cm = 4.2 cm

3. 2.3 cm – 1.9 cm = 0.4 cm

5.

7.

Lesson 3—Sound

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $v = (331 + (0.59/°\text{C})T)$ m/s
$= (331 + (0.59/°\text{C})(10.0°\text{C}))$ m/s
$= 337$ m/s

3. $v = \dfrac{d}{t}$
$d = vt$
$= (341 \text{ m/s})(5.50 \text{ s})$
$= 1.88 \times 10^3$ m

5. $v = (331 + (0.59/°\text{C})T)$ m/s
$= (331 + (0.59/°\text{C})(22°\text{C}))$ m/s
$= 344.0$ m/s
$v = \dfrac{d}{t}$
$d = vt$
$= (344.0 \text{ m/s})(3.6 \text{ s})$
$= 1.2 \times 10^3$ m

7. $v = (331 + (0.59/°\text{C})T)$ m/s
$= (331 + (0.59/°\text{C})(18.0°\text{C}))$ m/s
$= 342.0$ m/s
$v = \dfrac{d}{t}$
$t = \dfrac{d}{v}$
$= \dfrac{1.0 \times 10^4 \text{ m}}{342.0 \text{ m/s}}$
$= 29.2$ s

9. $v = (331 + (0.59/°\text{C})T)$ m/s
$355 = (331 + (0.59/°\text{C})T)$ m/s
$T = \left(\dfrac{355 - 331}{0.59}\right)°\text{C}$
$= 40.7°\text{C}$

11. $v = \lambda f$
$= (1.5 \text{ m})(1.00 \times 10^3 \text{ Hz})$
$= 1.5 \times 10^3$ m/s

13. $T = \dfrac{1}{f}$
$= \dfrac{1}{256 \text{ Hz}}$
$= 3.91 \times 10^{-3}$ s

Lesson 4—Characteristics of Sound

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $\lambda = 2L$
$= 2(1.2 \text{ m})$
$= 2.40$ m
$v = \lambda f$
$= (2.40 \text{ m})(65 \text{ Hz})$
$= 1.6 \times 10^2$ m/s

3. Third harmonic $= 3(\text{Fundamental frequency})$
$= 3(354 \text{ Hz})$
$= 1.06 \times 10^3$ Hz

5. Third harmonic $= 3(\text{Fundamental frequency})$

$$\text{Fundamental Frequency} = \frac{\text{3rd harmonic}}{3}$$
$$= \frac{1.2\times10^{3}\ \text{Hz}}{3}$$
$$= 4.00\times10^{2}\ \text{Hz}$$

$$\text{2nd harmonic} = 2(\text{Fundamental frequency})$$
$$= 2\left(4.00\times10^{2}\ \text{Hz}\right)$$
$$= 8.0\times10^{2}\ \text{Hz}$$

7. $\lambda = 2L$
$= 2(27.0\ \text{cm})$
$= 54.00\ \text{cm}$
$v = \lambda f$
$= (54.00\ \text{cm})(637\ \text{Hz})$
$= 3.440\times10^{4}\ \text{cm/s}$

Frequency of 22.0 cm string
$\lambda = 2L$
$= 2(22.0\ \text{cm})$
$= 44.00\ \text{cm}$
$f = \dfrac{v}{\lambda}$
$= \dfrac{3.440\times10^{4}\ \text{cm/s}}{44.00\ \text{cm}}$
$= 782\ \text{Hz}$

9. $v = \sqrt{\dfrac{T}{m/L}}$
$= \sqrt{\dfrac{23\ \text{N}}{1.9\times10^{-3}\ \text{kg}/0.80\ \text{m}}}$
$= 98\ \text{m/s}$

11. $v = \sqrt{\dfrac{T}{m/L}}$
$= \sqrt{\dfrac{80.0\ \text{N}}{195\times10^{-3}\ \text{kg}/5.0\ \text{m}}}$
$= 45.3\ \text{m/s}$
$\lambda = 2L$
$= 2(5.0\text{m})$
$= 10.0\ \text{m}$
$v = \lambda f$
$f = \dfrac{45.3\ \text{m/s}}{10.0\ \text{m}}$
$= 4.5\ \text{Hz}$

13. $v = \lambda f$
$v = \sqrt{\dfrac{T}{m/L}}$
$\therefore \lambda f = \sqrt{\dfrac{T}{m/L}}$
or $f \alpha \sqrt{T}$
or $T \alpha f^{2}$

Frequency
$\dfrac{\text{To}}{\text{From}} = \dfrac{231\ \text{Hz}}{224\ \text{Hz}}$
$= 1.031$
$f^{2} = 1.0635$
$T = \left(1.00\times10^{2}\ \text{N}\right)(1.0635)$
$= 1.06\times10^{2}\ \text{N}$

Lesson 5—Air Columns

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $v = \lambda f$
$\lambda = \dfrac{v}{f}$
$= \dfrac{343\ \text{m/s}}{256\ \text{Hz}}$
$= 1.34\ \text{m}$

3. $\text{3rd harmonic} = 3(\text{Fundamental Frequency})$
$= 3(384\ \text{Hz})$
$= 1.15\times10^{3}\ \text{Hz}$

5. $v = \left(331+(0.59/°\text{C})T\right)\ \text{m/s}$
$= \left(331+(0.59/°\text{C})(20.0°\text{C})\right)\ \text{m/s}$
$= 342.8\ \text{m/s}$
$v = \lambda f$
$\lambda = \dfrac{v}{f}$
$= \dfrac{342.8\ \text{m/ss}}{256\ \text{Hz}}$
$= 1.339\ \text{m}$
$L = \dfrac{\lambda}{4}$
$= \dfrac{1.339\ \text{m}}{4}$
$= 0.335\ \text{m}$

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7. $v = \lambda f$

$$\lambda = \frac{v}{f}$$
$$= \frac{341 \text{ m/s}}{4.40 \times 10^2 \text{ Hz}}$$
$$= 0.7750 \text{ m}$$
$$L = \frac{\lambda}{4}$$
$$= \frac{0.7750 \text{ m}}{4}$$
$$= 0.194 \text{ m}$$

9. $\lambda = 2L$
$= 2(0.330 \text{ m})$
$= 0.6600 \text{ m}$
$v = \lambda f$
$= (0.6600 \text{ m})(512 \text{ Hz})$
$= 338 \text{ m/s}$

11. 3rd harmonic
$3\text{rd harmonic} = 3(\text{Fundamental})$
$= 3(384 \text{ Hz})$
$= 1.15 \times 10^3 \text{ Hz}$

Lesson 6—Doppler Effect

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. Sound waves are being emitted from the train's whistle in all directions. The car travels toward these expanding wavefronts and the driver hears an apparent higher pitch than the source.

3. As the whistle moves toward the stationary observer, the apparent wavelength of the whistle wavefronts decreases (i.e., the wavefronts become compressed in front of the whistle). Thus, the observer hears an apparently higher frequency (pitch) sound than the source.

Practice Test

ANSWERS AND SOLUTIONS

1. The apparent change in frequency due to the motion of the source or the observer is called the Doppler effect.

C is the correct answer.

3. When waves meet crest to crest or trough to trough, they are meeting in phase. When waves meet in phase, constructive interference results.

A is the correct answer.

5. A wavelength is the distance between two adjacent points that are in phase. Points A and E are two such points.

C is the correct answer.

7. Each segment (loop) represents $\frac{1}{2}\lambda$. Therefore,

$$\frac{3}{2}\lambda = 4.5 \text{ m}$$
$$\lambda = 3.0 \text{ m}$$

$v = \lambda f$
$= (3.0 \text{ m})(60.0 \text{ Hz})$
$= 1.8 \times 10^2 \text{ m/s}$

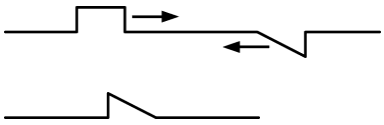
9. Resonance results when two objects have the same natural frequency, and the vibration of one of the objects causes the other object to vibrate.

D is the correct answer.

11. The amplitude is the maximum displacement from the equilibrium position. The amplitude is 10 cm.

C is the correct answer.

13.



A is the correct answer.

15. $f_b = f_2 - f_1$
$= 440 \text{ Hz} - 435 \text{ Hz}$
$= 5.0 \text{ Hz}$
The number of beats heard in 5.0 s can be calculated.
(5.0 Hz)(5.0 s) = 25 beats

17. A pure note of a given frequency has no overtones and vibrates only at the fundamental frequency. Sound coming from two different musical instruments will be recognizably different, even when the same note is played, because additional frequencies, called overtones, are produced. Thus, the quality of the sound produced by each instrument is different, allowing the instruments to be distinguished from one another.
An oscilloscope display of these two sounds would be very different because the display shows the superposition of these harmonics.

C is the correct answer.

19. First, express the time taken by the projectile and the sound in terms of their respective distances and speeds.

$$d_p = v_p t_p \Rightarrow t_p = \frac{d_p}{v_p}$$

$$d_s = v_s t_s \Rightarrow t_s = \frac{d_s}{v_s}$$

The difference in time is known.

$$\frac{d_s}{v_s} - \frac{d_p}{v_p} = 2.00 \text{ s}$$

The horizontal distance travelled by the projectile is the same as the distance travelled by the sound.

$$2.00 \text{ s} = \frac{d_p}{v_s} - \frac{d_p}{v_p}$$

$$2.00 \text{ s} = \frac{d_p}{340 \text{ m/s}} - \frac{d_p}{1\,250 \text{ m/s}}$$

$$2.00 \text{ s} = \left(\frac{(1\,250 \text{ m/s}) d_p}{(1\,250 \text{ m/s})(340 \text{ m/s})} - \frac{(340 \text{ m/s}) d_p}{(340 \text{ m/s})(1\,250 \text{ m/s})} \right)$$

$$2.00 \text{ s} = \frac{(1\,250 \text{ m/s}) d_p - (340 \text{ m/s}) d_p}{(1\,250 \text{ m/s})(340 \text{ m/s})}$$

$$2.00 \text{ s} = \frac{(910 \text{ m/s}) d_p}{(1\,250 \text{ m/s})(340 \text{ m/s})}$$

$$d_p = \frac{(1\,250 \text{ m/s})(340 \text{ m/s})}{(910 \text{ m/s})} \times 2.00 \text{ s}$$

$$d_p = 934 \text{ m}$$

21. $v = 331 \text{ m/s} + (0.59 \text{ m/s} \cdot °\text{C})(23°\text{C})$
$= 344.6 \text{ m/s}$

$$\lambda = \frac{v}{f}$$
$$= \frac{344.6 \text{ m/s}}{350 \text{ Hz}}$$
$$= 0.9846 \text{ m}$$

$$L = \frac{\lambda}{4}$$
$$= 0.246 \text{ m}$$

23. First, find the fundamental resonance.

$$L = \frac{\lambda}{4}$$
$$= \frac{145 \text{ cm}}{4}$$
$$= 36.3 \text{ cm}$$

Next, find the second harmonic.

$$L = \frac{3\lambda}{4}$$
$$= \frac{3(145 \text{ cm})}{4}$$
$$= 109 \text{ cm}$$

Finally, calculate the third harmonic.

$$L = \frac{5\lambda}{4}$$
$$= \frac{5(145 \text{ cm})}{4}$$
$$= 182 \text{ cm}$$

A is the correct answer.

25. The trumpet is a closed tube because it is closed at one end. An antinode always forms at the open end of a closed tube.

B is the correct answer.

27. The frequency of the *n*th harmonic is equal to *n* times the fundamental frequency.

$$nf_{\text{fund}} = (3)(40.0 \text{ Hz})$$
$$= 1.20 \times 10^2 \text{ Hz}$$

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29. $v = 331 \text{ m/s} + (0.59 \text{ m/s} \cdot °\text{C})(28°\text{C})$
$= 347.5 \text{ m/s}$
$L = \frac{\lambda}{4}$
$\lambda = 4(0.15 \text{ m})$
$= 0.600 \text{ m}$
$f = \frac{v}{\lambda}$
$= \frac{347.5 \text{ m/s}}{0.600 \text{ m}}$
$= 5.8 \times 10^2 \text{ Hz}$

31. Since the man is moving away from the source, it takes longer for each compression to reach him. The frequency he hears will be lower than *f*.

D is the correct answer.

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Student Notes and Problems

APPENDICES

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FORMULAS

KINEMATICS

$$v = \frac{\Delta d}{\Delta t} \qquad a = \frac{\Delta v}{\Delta t} \qquad d = \vec{v}t$$

$$v = v_0 + at \qquad \vec{v} = \frac{v + v_0}{2} \qquad d = v_0 t + \frac{1}{2}at^2$$

$$v^2 = {v_0}^2 + 2ad$$

FORCES AND DYNAMICS

$$F_{\text{g}} = mg \qquad F_{\text{g}} = G\frac{m_1 m_2}{r^2} \qquad F_{\text{f}} = \mu F_{\text{N}}$$

$$F = k\Delta x \qquad F_{\text{net}} = ma$$

ENERGY

$$W = Fd \qquad W = \Delta E \qquad W = Fd \qquad I = \frac{q}{t}$$

$$E_{\text{p}} = mgh \qquad E_{\text{k}} = \frac{1}{2}mv^2 \qquad V = IR \qquad \text{Efficiency} = \frac{W_{\text{out}}}{W_{\text{in}}} = \frac{P_{\text{out}}}{P_{\text{in}}}$$

$$P = IV \qquad P = I^2 R \qquad P = \frac{V^2}{R} \qquad P = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t} = Fv$$

Series Circuits	$I_T = I_1 = I_2 = I_3 = \ldots$	$V_T = V_1 + V_2 + V_3 + \ldots$	$R_{eq} = R_1 + R_2 + R_3 + \ldots$
Parallel Circuits	$I_T = I_1 + I_2 + I_3 + \ldots$	$V_T = V_1 = V_2 = V_3 = \ldots$	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$

WAVE MOTION AND GEOMETRICAL OPTICS

$$T = \frac{1}{f} \qquad v = \lambda f \qquad v = \left(331 + \left(0.59/^{\circ}\text{C}\right)T\right) \text{ m/s}$$

$$v = \sqrt{\frac{T}{m/L}} \qquad L = \frac{n\lambda}{2}$$

Beat frequency = $f_b = f_1 - f_2$

Closed-tube fundamental frequency
$\lambda = 4L$

Open-tube fundamental frequency
$\lambda = 2L$

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